4 Impulse and Impact

At the end of this section you should be able to:
a. define momentum and impulse
b. state principles of conservation of linear momentum
c. solve problems involving change and conservation of momentum

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4.1 Impulse of a force

The impulse of a constant force *F* is defined as the product of the force and the time *t* for which it acts.

$$\text{Impulse} = Ft$$

Equation 4.1

The effect of the impulse on a body can be found using Equation 1.1, where *a* is acceleration, *u* and *v* are initial and final velocities respectively and *t* is time.

$$v = u + at$$

So

$$\begin{aligned} mat &= m(v-u) \\ F &= ma \\ Ft &= m(v-u) = \text{change in momentum} \end{aligned}$$

Equation 4.2

So we can say that

$$\text{Impulse of a constant force} = Ft = \text{change in momentum produced}$$

Impulse is a vector quantity and has the sane units as momentum, *Ns* or *kg m/s*

The impulse of a variable force can be defined by the integral

$$\text{Impulse} = \int_0^t F\,dt$$

where t is the time for which *F* acts.

By Newton's 2nd law

$$F = ma = m\frac{dv}{dt}$$

So impulse can also be written

$$\begin{aligned} \text{Impulse} &= \int_0^t m\frac{dv}{dt}dt \\ &= \int_u^v mdv \\ &= [mv]_u^v \end{aligned}$$

Which for a constant mass

$$\text{Impulse} = m(v-u)$$

In summary

$$\text{Impulse} = \int_0^t F\,dt = \text{change in momentum produced}$$

Equation 4.3

4.2 Impulsive force

Suppose the force *F* is very large and acts for a very short time. Du the distance moved is very small and under normal analysis would be ignored. Under these condition the only effect of the force can be measured is the impulse, or change I momentum - the force is called an impulsive force.

In theory this force should be infinitely large and the time of action infinitely small. Some application where the conditions are approached are collision of snooker balls, a hammer hitting a nail or the impact of a bullet on a target.

Worked example 4.1

A nail of mass 0.02 kg is driven into a fixed wooden block, Its initial speed is 30 m/s and it is brought to rest in 5*ms*. Find

a) the impulse
b) value of the force (assume this constant) on the nail.

Solution

$$\begin{aligned}\text{Impulse} &= \text{change in momentum of the nail}\\ &= 0.02(30-0)\\ &= 0.6\,Ns\end{aligned}$$

From Equation 4.1

$$\text{Impulse} = Ft$$

$$F = \frac{\text{Impulse}}{t} = \frac{0.6}{0.005} = 120\,N$$

Worked Example 4.2

A football of mass 0.45 kg travels in a straight line along the ground reaching a player at 10m/s. The player passes it on at 8m/s altering its direction by 90°.
Find the impulse given to the ball by the player.

Solution

Choose the co-ordinate system like that in Figure 4.1

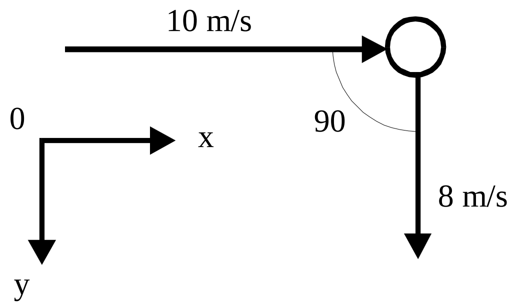

Figure 4.1: Co-ordinate system and path of ball in Worked Example 4.1

Initial velocity in the direction Ox is 10 m/s. Final velocity in the direction Ox is zero. So change in velocity in the direction Ox is -10m/s.

Initial velocity in the direction Oy ia zero and final velocity in the direction Oy is 8m/s. So change in velocity in the direc 8m/s.

The resultant change in velocity is

$$\sqrt{(-10)^2 + 8^2} = \sqrt{164} = 12.8\,m/s$$

impulse is change in momentum is mass times change in velocity

$$\text{Impulse} = m(v-u) = 0.45 \times 12.8 = 5.76\,Ns$$

4.3 Conservation of linear momentum

Consider the direct collision of two spheres A and B shown in Figure 4.3

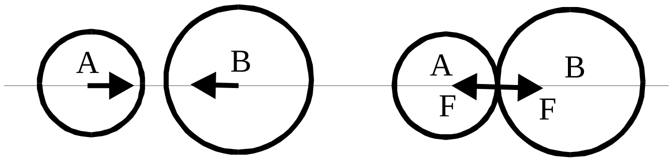

Figure 4.3: Direct collision of two spheres

When the spheres collide, then by Newton's third law, the force *F* exerted by A on B is equal and opposite to the force exerted by B on A.

The time for contact is the same for both. The impulse of A on B is thus equal and opposite to the impulse of B on A. It then follows that the change in momentum of A is equal in magnitude to the change in momentum in B - but it is in the opposite direction. The total change in momentum of the whole system is thus zero.

This means that the total momentum before and after a collision is equal, or that linear momen m is conserved. This is called **the principle of conservation of linear momentum** and in summary this may be stated:

The total momentum of a system, in any direction, remains constant unless an external force acts on the system in that direction.

4.4 *Impact of inelastic bodies*

When two inelastic bodies collide they remain together. They show no inclination to return to their original shape after the collision.

An example of this may be two railway carriages that collide and become coupled on impact.

Problems of this type may be solved by the principle of conservation of linear momentum.

$$\text{Momentum befor impact} = \text{Momentum after impact}$$

(this must be applied **in the same direction)**

Although momentum is conserved, it is important to realise that energy is always lost in an inelastic collision (it is converted from mechanical energy to some other form such as heat, light or sound.)

Worked Example 4.3

A railway wagon of mass 20 tonnes travelling at 1.5m/s collides with another of mass 30 tonnes travelling in the opposite direction at 0.5m/s. The wagons become coupled on impact. Find:

a) their common velocity after impact
b) the loss of kinetic energy.

Solution

a)

$$\text{Total momentum before impct} = \left(20\times10^3\times1.5\right) - \left(30\times10^3\times0.5\right) = 15\,000\ Ns$$

Note the negative sign for the second wagon as positive is taken as the direction of velocity of the 20 tonne wagon.

After the impact, is the common velocity is V then the momentum will be (20000 + 30000)V

using the conservation of linear momentum

$$\text{Momentum befor impact} = \text{Momentum after impact}$$
$$15000 = 50000V$$
$$V = 0.3\,m/s$$

This is positive, so it is in the original direction of the 20 tonne wagon.

b)

$$\text{kinetic energy before impact} = \frac{1}{2}20000\times1.5^2 + \frac{1}{2}30000\times0.5^2$$
$$= 26250\ J$$

$$\text{kinetic energy after impact} = \frac{1}{2}50000\times0.3^2 = 2250\ J$$

$$\text{loss of kinetic energy} = 26250 - 2250 = 24000\ J$$

Worked Example 4.4

A pile-driver of mass 2.5 tonnes drives a pile of mass 500 kg vertically into the gr d. The driver falls freely a vertical distance of 2m before hitting the pile and there is no rebound. Each blow of the drive moves the pile down 0.2m. What is the average value of resistance of the ground to penetration?

Solution

The velocity of the pile-driver just before it hits the pile can be found using Equation 1.4

$$v^2 = u^2 + 2as$$

u = 0.0, a = 9.81m/s^2, s = 2m

$$v^2 = 0.0 + 2 \times 9.81 \times 2$$

$$v = \sqrt{39.24} = 6.26\, m/s$$

The momentum just before impact it thus

$$\text{momentum before impact} = 2.5 \times 10^3 \times 6.26$$

$$= 15650\, Ns$$

Since there is no rebound, the pile and driver have the same velocity after impact. So we can write this expression for momentum after impact if the common velocity is V:

$$\text{momentum after impact} = (2500 + 500)V = 3000V$$

So by the principle of conservation of momentum

$$\text{Momentum befor impact} = \text{Momentum after impact}$$

$$15650 = 3000V$$

$$V = 5.22\, m/s$$

The pile and driver are now brought to rest by the deceleration force of the ground in 0.2m. we can find this deceleration using Equation 1.4

$$v^2 = u^2 + 2as$$

u = 5.22 m/s, s = 0.2m, v = 0.0

$$0.0 = 5.22^2 + 2a0.2$$

$$a = -68.1\, m/s^2$$

Now the retarding force is given by

$$F = ma$$

$$F = 3000 \times 68.1$$

$$F = 204300\, N$$

the ground resistance, R, is the sum of this retarding force and the weight of the pile and driver

$$R = 3000 \times 9.18 + 204300 = 233730\, N$$

4.5 Impact of elastic bodies

In the last section the bodies were assumed to stay together after impact. An elastic body is one which tends to return to its original shape after impact. When two elastic bodies collide, they rebound after lision. An example is the collision of two snooker balls.

If the bodies are travelling along the same straight line before impact, then the collision is called a direct collision. This is the only type of collision considered here.

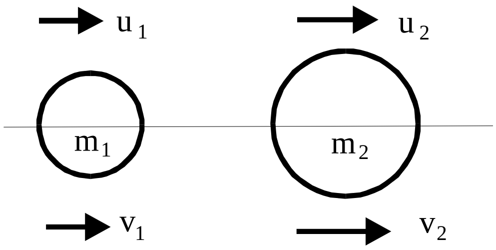

Figure 4.4: Direct collision of two **elastic** spheres

Consider the two elastic spheres as shown in Figure 4.4. By the principle of conservation of linear momentum

$$\text{Momentum befor impact} = \text{Momentum after impact}$$
$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

Equation 4.4

Where the *u*'s are the velocities before collision and the *v*'s, the velocities after.

When the spheres are inelastic v_1 and v_2 are equal as we saw in the last section. For elastic bodies v_1 and v_2 depend on the elastic properties of the bodies. A measure of the elasticity is the **coefficient of restitution** *e*, For direct collision this is defined as

$$e = -\left(\frac{v_1 - v_2}{u_1 - u_2}\right)$$

Equation 4.5

This equation is the result of experiments performed by Newton.

The values of *e* in practice vary from between 0 and 1. For inelastic bodies $e = 0$, for completely elastic $e = 1$. in this latter case no energy is lost in the collision.

Worked Example 4.5

A body of mass 2kg moving with speed 5m/s collides directly with another of mass 3 kg moving in the same direction. The coefficient of restitution is 2/3. Find the velocities after collision.

Solution

$$\text{Momentum befor impact} = \text{Momentum after impact}$$
$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$
$$2 \times 5 + 3 \times 4 = 2v_1 + 3v_2$$
$$22 = 2v_1 + 3v_2$$

[1]

By Equation 4.5

$$e = -\left(\frac{v_1 - v_2}{u_1 - u_2}\right)$$
$$\frac{2}{3} = -\left(\frac{v_1 - v_2}{5 - 4}\right)$$
$$-2 = 3v_1 - 3v_2$$

[2]

Adding [1] and [2] gives