Fundamentals of Mechanics - CIVE 1140

Kinetics Examples

Section 4: Impulse and impact

4.1 A nail of mass 0.01 kg travelling horizontally at 10m/s as it enters a fixed wooden block is brought to rest in 1ms. Find the average resistance of the wood.
(100 N)

$$m = 0.01 \text{ kg}$$
$$u = 10 \text{ m/s}$$
$$s = 1 \text{ m}$$
$$a = (v-u)/t$$

$$\text{Impulse} = Ft = m(v-u) = 0.01 \times 10 = 0.1$$

$$Ft = 0.1$$
$$F = 0.1/0.001 = 100 \text{ N}$$

4.2 A railway wagon of mass 5 tonnes strikes a pair of buffers at 2m/s and rebounds at 1.5m/s. The time of contact with the buffers is 0.4s. What is the average force exerted by the buffers?
(43.75 kN)

$$m = 5000 \text{ kg}$$
$$u = 2 \text{ m/s}$$
$$v = -1.5 \text{ m/s}$$
$$t = 0.4\text{s}$$

$$Ft = m(v-u) = 5000\ (-1.5 - 2\ ) = 17500$$
$$F = 17500/0.4 = 43750 \text{ N}$$

4.3 A 5kg mass falls freely from a height of 2m onto a fixed horizontal surface. It rebounds to a height of 0.5m.The time of impact is 0.025s. Find a) the change in momentum and the average force between the mass and the surface.
(46.98 Ns, 1878 N)

$$m = 5 \text{ kg}$$
$$h = 2\text{m} , 0.5\text{m}$$
$$t = 0.025$$

initial part of motion

$$v^2 = u^2 + 2as$$
$$v^2 = 2 \times 9.81 \times 2$$
$$v = 6.26 \text{ m/s}$$

rebound part of motion

$$v^2 = u^2 + 2as$$
$$0 = u^2 - 2 \times 9.81 \times 0.5$$
$$u = -3.132 \text{ m/s}$$

$$Ft = m\ (v - u) = 5\ (\ 6.26 - (-3.132)\ ) = 46.98$$
$$F = 46.98/0.025 = 1879 \text{ N}$$

4.4 A pile of mass 0.5 tonnes is driven vertically into the ground by a pile-driver of mass 1.2 tonnes. The driver falls freely 4m before striking the pile, and the ground resistance is a constant 150 kN. Find how far the pile is driven into the ground if there is no rebound.
(0.25 m)

$$m_1 = 500 \text{ kg}$$
$$m_2 = 1200 \text{ kg}$$
$$F = 150\,000 \text{ N}$$

$$v^2 = u^2 + 2as$$
$$v^2 = 2 \times 9.81 \times 4$$
$$v = 8.85 \text{ m/s}$$

$$1200 \times 8.85 = 10630 = 1700 \text{ v}$$
$$v = 6.26 \text{ m/s}$$

$$F = 1500 - 1700 \times 9.81 = 133\,323 \text{ N}$$

$$Ft = 10630$$
$$t = 10630 / 133\,323 = 0.0797 \text{ s}$$

$$s = 0.5(v+u) \times t = 6.25/2 \times 0.0797 = 0.25 \text{ m}$$

4.5 A body of mass 1kg moving at 3.5 m/s collides with another body of mass 2.5kg moving in the same direction along the same line at 1m/s. If the coefficient of restitution is 0.8, find the velocities of the bodies after impact.
(0.286 m/s, 2.286 m/s, both in the original directions)

$$m_1 = 1 \text{ kg}$$
$$u_1 = 3.5 \text{ m/s}$$
$$m_2 = 2.5 \text{ kg}$$
$$u_2 = 1 \text{ m/s}$$
$$e = 0.8$$

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$
$$3.5 = 2.5 = v_1 + 2.5\ v_2$$
$$6 = v_1 + 2.5\ v_2$$

eqn 1

$$e = -(v_1 - v_2)/(u_1 - u_2) = 0.8 = (v_1 - v_2)/(-2.5)$$

$$-2.0 = v_1 - v_2$$

eqn 2

eqn 1 - eqn 2
$$8 = 3.5\ v_2$$

$$v_2 = 2.286 \text{ m/s}$$

$$v_1 = -2 + 2.286 = 0.286 \text{ m/s}$$

4.6 A hammer of mass 1 kg, moving horizontally at 5m/s, strikes a nail of mass 0.025kg and drives it 25mm into a vertical wooden plank. Assuming there is no rebound, calculate a) the common speed after impact, b) the time of motion of the nail, c) the average resistance of the wood and d) the loss of kinetic energy.
(4.878 m/s, 0.010s, 488 N, 0.295 J)

$$m_1 = 1 \text{ kg}$$
$$u_1 = 5 \text{ m/s}$$
$$m_2 = 0.025 \text{ kg}$$
$$s = 0.025 \text{ m}$$

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$
$$v_1 = v_2 = v$$

$$5 + 0 = 1.025 \text{ v}$$

a)

$$v = 4.878 \text{ m/s}$$

b)

$$s = 0.5(u+v) \times t$$
$$0.025 = 0.5 \times 4.878\ t$$
$$t = 0.010 \text{ s}$$

c)

$$Ft = m\,(v - u) = 1.025 \times 4.878 = 5$$
$$F = 5 / 0.01 = 500 \text{ N}$$

d)

$$Ke = mv^2/2 = 0.025 \times 4.878^2/2 = 0.297 \text{ J}$$

4.7 A railway wagon of mass 8 tonnes rolls from rest down an incline 1km long which falls 1m in 100m along the track. The resistance to motion is a constant 500N. Find the speed of the wagon at the bottom of the incline. When it reaches the bottom of the incline the wagon collides, inelestically with a second wagon of mass 10 tonnes and both move off together horizontally. Find their common speed after the collision.
(8.44 m/s, 3.75 m/s)

$$m_1 = 8000 \text{ kg}$$
$$s = 1000\text{m}$$
$$R = 500 \text{ N}$$

$$F = 8000 \times 9.81 \times (1/100) - 500 = 284 \text{ N}$$

$$F = ma$$
$$a = 284/8000 = 0.0356$$

$$v^2 = u^2 + 2as$$
$$v^2 = 2 \times 0.0356 \times 1000$$
$$v = 8.438 \text{ m/s}$$

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$
$$v_1 = v_2 = v$$

$$8000 \times 8.438 = 18000\ v$$
$$v = 3.75 \text{ m/s}$$

4.8 A railway wagon of mass 20 tonnes, moving at 3 m/s, collides with a second wagon of mass 15 tonnes which is stationary. The coefficient of restitution if 0.6. Find the loss of kinetic energy during the impact.
(24.7 kJ)

$$m_1 = 20\,000 \text{ kg}$$
$$u_1 = 3 \text{ m/s}$$
$$m_2 = 15\,000 \text{ kg}$$
$$u_2 = 0$$
$$e = 0.6$$

$$20000 \times 3 + 0 = 20000\ v_1 + 15000\ v_2$$
$$60 = 20v_1 + 15v_2$$
$$0.6 = -(v_1 - v_2)/3$$

$$-1.8 = v_1 - v_2$$
$$-36.36 = 20v_1 - 20v_2$$

$$96.36 = 35\ v_2$$

$$v_2 = 2.753 \text{ m/s}$$
$$v_1 = 0.952 \text{ m/s}$$

$$Ke = 0.5 \times 20000 \times 0.952^2 + 0.5 \times 15000 \times 2.753^2 = 65905 \text{ J}$$
$$Ke \text{ before} = 0.5 \times 20000 \times 3^2 = 90000 \text{ J}$$

$$\text{Loss in Ke} = 24095 \text{ J}$$