Mechanics Example Sheet 4 – Dec 2002

Solutions

1.a A wheel starts at rest and uniformly reaches a speed of 2000 rev/min in 2.0s. What is its angular acceleration? The wheel has diameter 300mm, what is the final linear speed of a point on its edge?

1.b An electric train has total mass of 350 tonnes and has a tractive force is 150 kN. If the frictional force of the track is a constant 50 kN, find the acceleration a) up an incline of 1 in 200 (along the incline) and b) down an incline of 1 in 150 (along the incline.)

(4 marks)

1.a

Convert the units

$$r = 300/2\,mm = 0.15m$$
$$n = 2000 rev/\min$$
$$= \frac{2000}{60}$$
$$= 33.3\,rev/s$$

Angular velocity

$$\omega = 2\pi m = 209.4$$

Angular acceleration

$$\alpha = \frac{\omega_2 - \omega_1}{t} = 104.72$$

Linear speed

$$v = r\omega$$
$$= 0.15 \times 209.4$$
$$= 31.41 m/s$$

1.b part a

$$m = 350\,tonnes = 350\,000 kg$$
$$F = 150\,kN = 150000\,N$$
$$Fr = 50\,kN = 50000\,N$$

$$F_{\text{resultant}} = 150000 - 50000 - mg\sin\theta$$
$$= 100000 - 350000 \times 9.81 \times \frac{1}{200} = 82833\,N$$
$$F = ma$$
$$a = \frac{82833}{350000} = 0.237\,m/s^2$$

part b

$$F_{\text{resultant}} = 150000 - 50000 + mg\sin\theta$$
$$= 100000 + 350000 \times 9.81 \times \frac{1}{150} = 122890\,N$$

$$F = ma$$
$$a = \frac{122890}{350000} = 0.35\,m/s^2$$

2.a The force on a body varies with distance as shown in this table:

Force (kN)	0	7	1	35	30	28	17
Distance moved(m)	0	5	20	40	50	60	100

Find the work done as the body moves 100m

2.b A ball of mass 1kg moving at 3.0 m/s collides with another ball of mass 2.0kg moving in the same direction along the same line at 1.5m/s. If the coefficient of restitution is 0.9, find the velocities of the balls after impact.

(4 marks)

2a

Work done is area under the force / distance curve

$$\text{Area} = \left(\frac{7}{2}\right)5 + \left(\frac{7+1}{2}\right)15 + \left(\frac{35+1}{2}\right)20 + \left(\frac{30+35}{2}\right)10 + \left(\frac{30+28}{2}\right)10 + \left(\frac{28+17}{2}\right)40 = 1952.5\,kJ$$

2b

$$m_1 = 1kg \qquad u_1 = 3m/s$$
$$m_1 = 2\,kg \qquad u_1 = 1.5m/s$$
$$t = 0.9s$$

By conservation of momentum

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$
$$3 + 3 = v_1 + 2v_2$$

eqn 1

Coefficient of restitution

$$\varepsilon = -\left(\frac{v_1 - v_2}{u_1 - u_2}\right)$$
$$0.9(3 - 1.5) = -v_1 + v_2$$

eqn 2

Add eqn 1 + eqn 2

$$7.35 = 3v_2$$
$$v_2 = 2.45m/s$$
$$v_1 = 1.1m/s$$

3 A 3kg body is projected up a rough slope at 30° to the horizontal with an initial kinetic energy of 400 J. The coefficient of friction is 0.25. Find: a) the retarding force; b) the deceleration; c) the distance travelled before stopping and d) the loss in energy due to friction

(4 marks)

$$m = 3kg$$
$$\mu = 0.25$$
$$ke_{\text{initial}} = 400\,J$$

$$F = mg\sin 30$$
$$= 3 \times 9.81 \times 0.5 = 14.715\,N$$
$$Fr = \mu N$$
$$N = mg\cos 30 = 25.49\,N$$
$$Fr = 6.37\,N$$

a)

$$\text{Effective force} = 14.715 + 6.37 = 21.085\,N$$

b)

$$F = ma$$
$$a = \frac{21.085}{3} = 7.03 m/s^2$$

c)

$$ke_{\text{initial}} = \frac{1}{2}mu^2 = 400\,J$$
$$400 = \frac{1}{2}3u^2$$
$$u = 16.33\,m/s$$

$$v^2 = u^2 + 2as$$
$$0 = 16.33^2 - 2 \times 7.03s$$
$$s = 18.98\,m$$

d)

Calculate height risen, *h*, when travelling 18.98 m along the slope

$$\sin 30 = \frac{h}{s}$$
$$h = 9.485\,m$$
$$Pe_{\text{initial}} = 0$$
$$Pe_{\text{final}} = mgh = 279.14\,J$$
$$ke_{\text{final}} = 0$$
$$\text{Loss in energy} = 400 - 279.14 = 120.86\text{J}$$

4 A tank, of cross-sectional area 8 m^2, contains water at a depth of 5m. The water is pumped out in 1.5 hours at 4m/s to a similar sized tank whose base is 20m higher than the base of the first tank.
Taking the density of water as 1000 kg/m^3, find:

i) the potential energy given to the water;

ii) the kinetic energy given to the water

iii) the average power of the pump.

$$A = 8\ m^2$$
$$\text{Volume of water} = 8 \times 5 = 40\ m^3$$
$$\text{Mass} = 40 \times 1000 = 40\ 000\ kg$$
$$\text{Time} = 1.5h = 1.5 \times 60 \times 60 = 5400\ s$$
$$v = 4m/s$$

i)

$$Pe = mgh = 40\ 000 \times 9.81 \times 20 = 7848\ 000\ J = 7848\ KJ = 7.85\ MJ$$

ii)

$$Ke = mv^2/2 = 40\ 000 \times 4^2/2 = 320\ 000\ J = 320\ KJ$$

ii)

$$P = Fv$$

$$F = ma = 40\ 000 \times 9.81 = 392\ 400\ N$$

$$v = s/t = 20/5400 = 0.0037\ m/s$$

$$P = 392400 \times 0.0037 = 1451.88\ W$$

Or without loosing the accuracy in the velocity calculation

$$P = 392400 \times 20 / 5400 = 1453.33\ W$$