CIVE 1400-

May/June 2000

Examination for the degree of

BEng/MEng
Civil Engineering

FLUID MECHANICS

Time allowed: 2 hours

Attempt 4 questions

Useful formulae:

Parallel axis theorem $I_{oo} = I_{GG} - A\bar{x}^2$, 2nd moment of area of a rectangle $I_{GG} = \frac{bd^3}{12}$

1 A 5*m* wide tank with an L-shaped cross section, as shown in Figure 1, has a gate which is hinged at the top at it right hand end. If the tank is filled with water to a level of 8*m* determine the torque required at the hinge to just keep the gate closed.

(20 marks)

Determine also the force on the base of the tank and comment on why this is not the same as the weight of the water.

(5 marks)

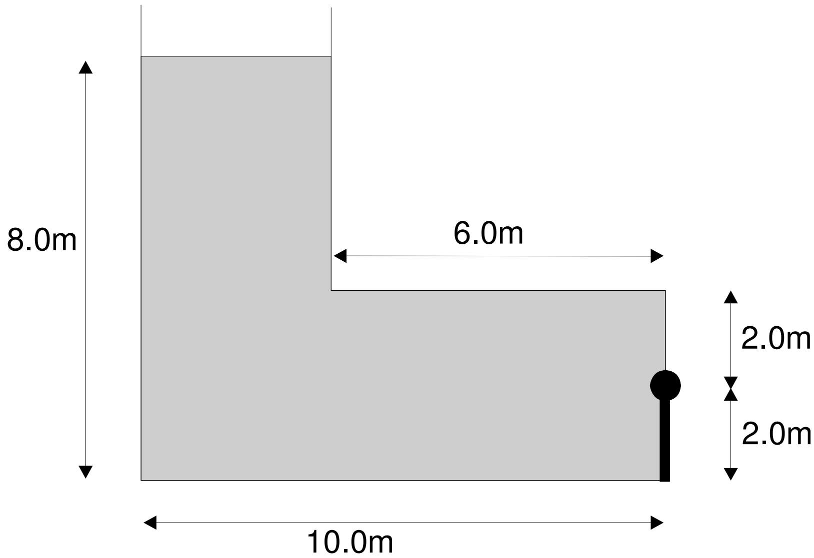
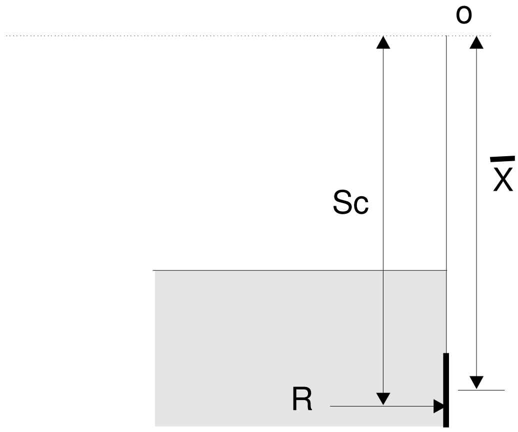


Figure 1

$$\text{Force on gate} = \text{pressure at centroid} \times \text{area of gate}$$
$$= \rho g(6+1)\times(2\times 5)$$
$$= 9810 \times 7 \times 10$$
$$= 686\,700\ N$$

Position of action of force = Sc from the point O

The point O is the point where a line from the gate crosses the top water surface level



Position of this force is through the centre of pressure, Sc.
Using the parallel axis theorem,

$$Sc = \frac{I_{oo}}{A\overline{x}} = \frac{\text{2nd momnt of area}}{\text{1st moment of area}}$$

$$I_{oo} = I_{GG} - A\overline{x}^2$$

$$Sc = \frac{I_{GG}}{A\overline{x}} + \overline{x}$$

$\overline{x}$ is the distance along the face to the centroid

$$I_{GG} = \frac{bd^3}{12} = \frac{5 \times 2^2}{12} = 3.333$$

$$Sc = \frac{bd^3}{12}\frac{1}{bd}\frac{1}{7} + 7 = \frac{4}{84} + 7 = 7.0476m$$

Torque required = $(7.0476 - 6) \times 686700 = 719400$ Nm clockwise

Force on base = PA = pghA = 3924 kN

Weight of water = (10×8×5 - 4×6×5)×9810 = 2746800 N

2 Water flows along a circular pipe and is turned vertically through 180° by a reducing bend as shown in figure 2. The rate of flow in the pipe is 20 litres/s, the pressure measured at the entrance to the bend is 120 kN/m^3 and the volume of fluid in the bend is 0.1m^3. What is the magnitude and direction of the force exerted by the fluid on the bend? Ignore any friction losses.

(25 marks)

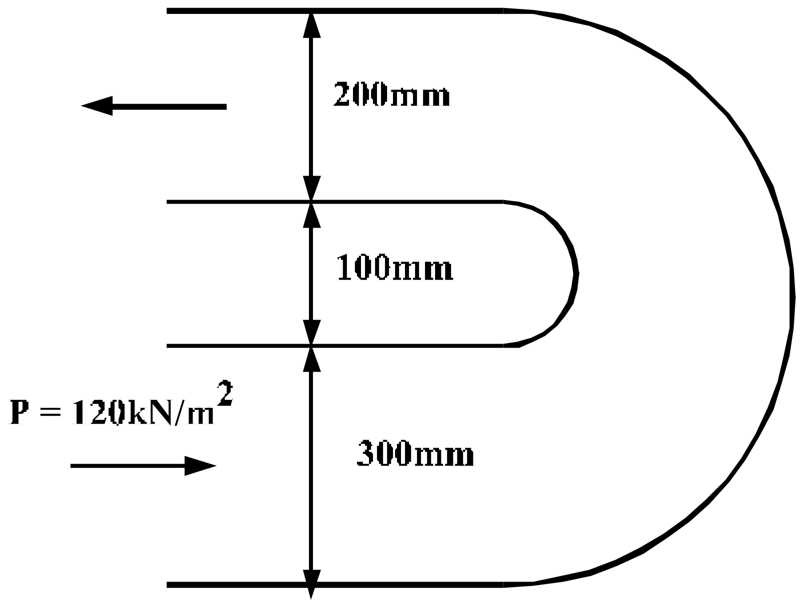


Figure 2

Take the inlet as point 1 and the outlet as point 2.

$$Q = 2 \text{ l/s} = 0.02 \text{ m}^3/\text{s}$$
$$p_1 = 120000 \text{ N/m}^2$$

Height difference between the two pipe centres,

$$h = 0.35\text{m}$$

$p_1 = 120000$ N/m^2 $\qquad$ $p_2 = ?$

$d_1 = 0.3$m $\qquad$ $d_2 = 0.2$ m

$$a_1 = \frac{\pi d_1^2}{4} = 0.0707\, m^2 \qquad a_2 = \frac{\pi d_2^2}{4} = 0.0314\, m^2$$

$$Q = a_1 u_1 = a_2 u_2$$

$$u_1 = \frac{0.02}{0.0707} = 0.2829\, m/s$$

$$u_2 = \frac{0.02}{0.0314} = 0.6369\, m/s$$

<u>Calculate the total force on the bend.</u>

In the x- direction

$$\begin{aligned} F_{Tx} &= \rho Q(u_{2x} - u_{1x}) \\ &= \rho Q(u_2 \cos\theta - u_1) \\ &= 1000 \times 0.02(-0.6369 - 0.2829) \\ &= -18.40\, N \end{aligned}$$

In the y-direction there is no component i.e.

$$F_{Ty} = \rho Q\left(u_{2y} - u_{1y}\right) = 0$$

Calculate the pressure force

F_p = pressure force at 1 - pressure force at 2

$$\begin{aligned} F_{px} &= p_1 A_1 \cos 0 - p_2 A_2 \cos 180 \\ &= p_1 A_1 + p_2 A_2 \end{aligned}$$

$$F_{py} = 0$$

We know p_1 but need to find p_2 using Bernoulli from point 1 to point 2.

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2$$

$$\frac{12000}{9810} + \frac{0.2829^2}{19.62} + 0 = \frac{p_2}{9810} + \frac{0.6369^2}{19.62} + 0.35$$

$$p_2 = 161404\, N/m^2$$

$$\begin{aligned} F_{px} &= p_1 A_1 + p_2 A_2 \\ &= 120000 \times 0.0707 + 161404 \times 0.0314 \\ &= 121369 N \end{aligned}$$

Calculate the body forces

The only body force is that due to gravity i.e the weight of the water.
We are given the volume = 0.1 m^3

$$\begin{aligned} F_{By} &= \rho g \times volume \\ &= 981\, N \end{aligned}$$

$$F_{Bx} = 0$$

Calculate the resultant force acting on the fluid

$$\begin{aligned} F_{Tx} &= F_{Rx} + F_{Px} + F_{Bx} \\ -18.40 &= F_{Rx} + 12139 \\ F_{Rx} &= 12157\, N \end{aligned}$$

$$\begin{aligned} F_{Ty} &= F_{Ry} + F_{Py} + F_{By} \\ 0 &= F_{Ry} + 0 - 981 \\ F_{Ry} &= 981\, N \end{aligned}$$

$$F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2} = 12197\,N$$

Acting at the angle θ

$$\tan\theta = \frac{F_{Ry}}{F_{Rx}}$$
$$\theta = 4.61'$$

The force acting on the bend is -12197 N i.e. equal in magnitude to the force on the fluid, but in the opposite direction.

3 Describe with the aid of diagrams the following phenomena explaining why and when they occur.
(Each part requires at least a half page description of the phenomenon plus diagrams.)

(i) The laminar boundary layer
(5 marks)

(ii) The turbulent boundary layer
(5 marks)

(iii) The laminar sublayer
(5 marks)

(iv) Boundary layer separation
(5 marks)

(v) Methods to prevent boundary layer separation
(5 marks)

Answer: As the question says - EACH PART REQUIRES AT LEAST HALF A PAGE DESCRIPTION PLUS DIAGRAMS - take from lecture notes AND other books.

4.a Starting from the Bernoulli equation, develop the equation shown below for discharge over a sharp edged rectangular weir. State **all** assumptions made.

$$Q_{\text{actual}} = C_d \frac{2}{3} B\sqrt{2g} H^{3/2}$$

(15 marks)

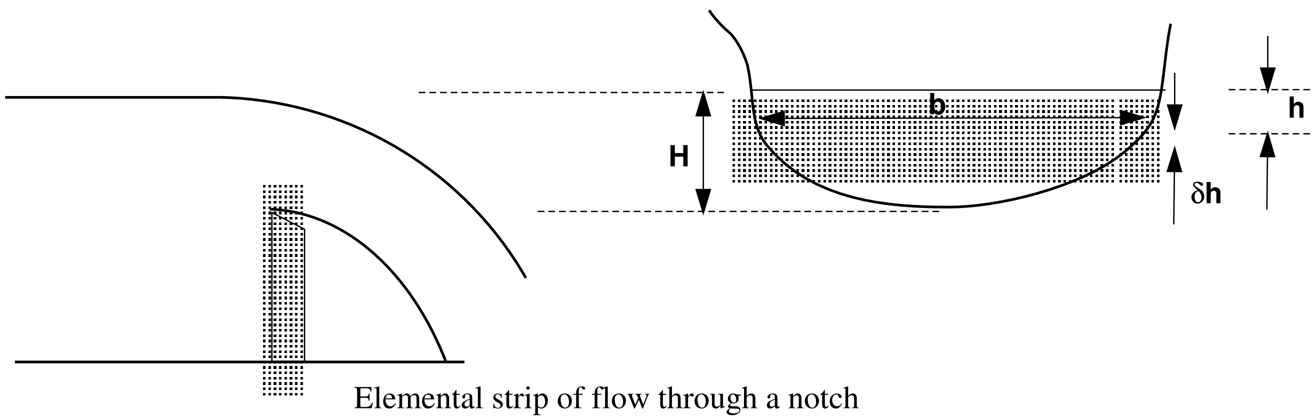
4.b At the end of a channel is a sharp edged rectangular weir with a width of 400mm and a coefficient of discharge of 0.65. The water is flowing at a depth 0.16m above the base of the weir. If this weir is replaced by a 90° V-notch weir with the same coefficient of discharge, what will be the necessary upstream depth of water to achieve the same discharge as the rectangular weir.
The equation for discharge over a v-notch weir is:

$$Q_{\text{actual}} = C_d \frac{8}{15}\sqrt{2g}\tan\left(\frac{\theta}{2}\right) H^{5/2}$$

(10 marks)

4.a

A General Weir Equation
Consider a horizontal strip of width *b* and depth *h* below the free surface, as shown in the figure below.



Elemental strip of flow through a notch

Assuming the velocity is only due to the head i.e. a very slow flow towards the weir. Then from the Bernoulli equation we get this expression for:

$$\text{velocity through the strip, } u = \sqrt{2gh}$$

$$\text{discharge through the strip, } \delta Q = Au = b\delta h\sqrt{2gh}$$

Integrating from the free surface, $h = 0$, to the weir crest, $h = H$ gives the expression for the total theoretical discharge

$$Q_{\text{theoretical}} = \sqrt{2g}\int_0^H bh^{1/2}\, dh$$

This will be different for every differently shaped weir or notch. To make further use of this equation we need an expression relating the width of flow across the weir to the depth below the free surface.

For a rectangular weir b = B = constant
The actual discharge is obtained by introducing a coefficient of discharge

$$Q_{\text{theoretical}} = \sqrt{2g}\int_0^H Bh^{1/2}\,dh$$

$$Q_{actual} = Cd\,\frac{2}{3}\sqrt{2g}\,H^{3/2}$$

For the rectangular weir

$$\begin{aligned} Q &= 0.65\times 0.4\times\frac{2}{3}\times\sqrt{19.62}\times 0.16^{3/2} \\ &= 0.049\ m^3/s \end{aligned}$$

For the v-notch weir

$$\begin{aligned} 0.049 &= 0.65\times\frac{8}{15}\times\sqrt{19.62}\times\tan 45\times H^{5/2} \\ 0.049 &= 1.5355\,H^{5/2} \\ H^{5/2} &= 0.0319 \\ H &= 0.252\,m \end{aligned}$$

(Note you have to use the inverse-root function of your calculator to get the value for H from $H^{5/2}$)

5 A rectangular sluice gate is fitted at the base of a reservoir wall with a pivot in the arrangement shown in Figure 3. The gate is designed to regulate the level of water in the reservoir by opening when the water level to the right, h, reaches a certain depth. The gate has a width of 1.2m and its centre of gravity is 0.3m from the wall. Determine the weight, W, of the gate, if a water level of h = 2.779m will just cause the gate to open.

(25 marks)

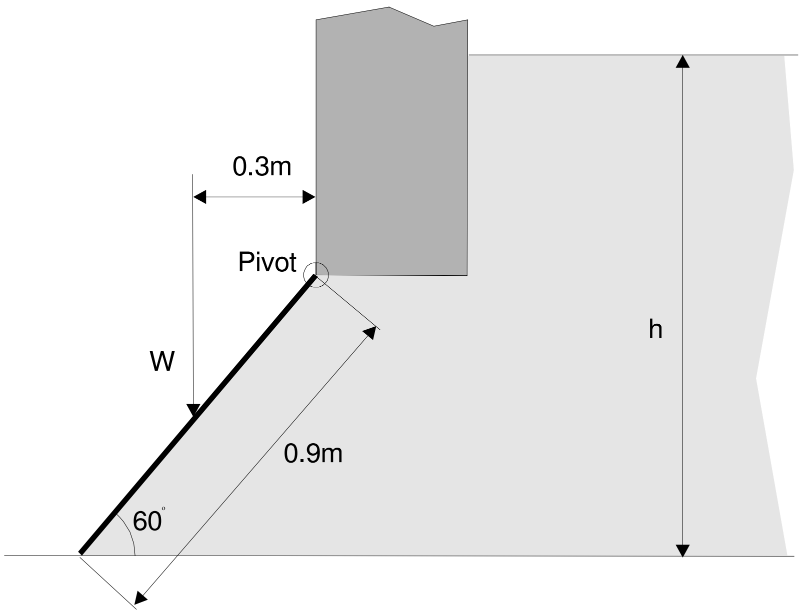


Figure 3

$L = 0.9m$

$x = 0.9\cos 60 = 0.45m$

$y = 0.9\sin 60 = 0.779m$

The gate opens when the moment at the pivot is clockwise.
That is when the moment due to the water > 0.3w.

Method 1

Force on plane = Area × Pressure at centroid

$$\text{Force} = (1.2 \times 0.9) \times \left(2.779 - \frac{y}{2}\right)\rho g$$
$$= 1.08 \times (2.779 - 0.779/2)1000 \times 9.81$$
$$= 25316N$$

Method 2

Horizontal force = Hf = Area of projection on vertical plane × pressure at centroid

$$H_f = 1.2y\left(2.779 - \frac{y}{2}\right)\rho g$$
$$= 21912.6N$$

Vertical force = weight of water above gate

$$V_f = \left(2.779x - \frac{x \times y}{2} \right) \rho g 1.2$$
$$= (12268 - 1719)1.2$$
$$= 12658$$

Resultant force

$$R = \sqrt{H_f^{\ 2} + V_f^{\ 2}}$$
$$= 25306\,N$$

Method 3

$$\text{Force} = \text{Shaded area}$$
$$= \left(2.779 \times 0.9 - \frac{0.779 \times 0.9}{2} \right) 1.2 \rho g$$
$$= 2.15055 \times 1.2 \rho g$$
$$= 25316\,N$$

Point of action of the force = centre of pressure

$$x_2 = \frac{2}{\tan 60} = 3.444m$$
$$L_2 = \frac{2}{\sin 60} = 2.309m$$
$$\bar{x} = L_2 + \frac{0.9}{2} = 2.759m$$

$$Sc = \frac{\text{2nd moment of area about O}}{\text{1st moment of area about O}} = \frac{I_{oo}}{A\bar{x}}$$
$$I_{oo} = I_{GG} + A\bar{x}^2$$
$$Sc = \frac{I_{GG}}{A\bar{x}} + \bar{x}$$

For a rectangle, $I_{GG} = \frac{bd^3}{12}$

$$Sc = \frac{d^2}{12\bar{x}} + \bar{x}$$
$$= \frac{0.9^2}{12 \times 2.759} + 2.759$$
$$= 2.783m$$

Need to find the lever arm, i.e. the distance from the pivot to the centre of pressure specified by Sc.
First find the position of the pivot, x_2, from the surface (along the inclined plane)

$$x_2 \cos 30 = 2$$
$$x_2 = 2.309\text{m}$$

Lever are, x_1 = Sc - x_2 = 2.783 - 2.309 = 0.474 m

Take moments to fine the weight of the gate, w

$$Rx_1 = 0.3w$$
$$w = \frac{25316 \times 0.474}{0.3} = 40000\,N$$

6.a Describe what is meant by the term dimensional analysis. Your explanation should include the meanings and relevance of the terms *geometric similarity, dynamic similarity* and *kinematic similarity* as well as identifying some uses form the technique.

(8 marks)

6.b Assuming the drag force, F, exerted on a body is a function of the following

fluid density	ρ
fluid viscosity	μ
diameter	d
velocity	u

Show that the drag force can be expressed as

$$F = d^2 u^2 \rho\ \phi(\mathrm{Re})$$

where ϕ is some unknown function and Re is the Reynolds number.

(10 marks)

6.c A prototype boat propeller has a diameter of 1.0m. It is necessary to determine the force it will experience when water flows past at 5 m/s. A model propeller is available of diameter 0.1m and can be placed in a wind tunnel. To obtain the dynamically similar conditions at what velocity would the air need to flow in the wind tunnel?

(7 marks)

$\mu_{water} = 1.0 \times 10^{6}$ kg/ms $\qquad \mu_{air} = 1.7 \times 10^{5}$ kg/ms

$\rho_{water} = 1000$ kg/m^3 $\qquad \rho_{air} = 12.5$ kg/m^3

6.a.

Dimensional analysis is used when constructing physical models of prototype structures. Physical models are used when the fluid flow is particularly complex and difficult to analyse by other means. It enables physical measurements - forces, velocities etc. - taken from the scale models to be converted to the equivalent measurement which would be found on a prototype.

The term similarity relates to physical a scale models.
Geometric similarity - all dimensions are in the in the same ratio.
Dynamic similarity - all velocities are in the same ratio - requires geometric similarity
Kinematic similarity - all forces are in the same ration - requires dynamic similarity.

6.b.

$$F = f(\rho, \mu, d, u)$$
$$0 = \phi(F, \rho, \mu, d, u)$$

Assume the governing variables ρ, *u, d*

According to Buckingham's π theorem there are n-m groups
where

n = number of variables (5) and
m = number of dimensions (i.e. MLT, giving 3)
n-m = 5-3 = 2 groups

$$0 = \phi(\pi_1, \pi_2)$$
$$\pi_1 = \rho^{a_1} u^{b_1} d^{c_1} \mu$$
$$\pi_2 = \rho^{a_2} u^{b_2} d^{c_2} F$$

Dimensions of the variables are:

ρ = density (kg/m^3) = ML^{-3}
μ = viscosity (kg/m/s) = $ML^{-1}\, T^{-1}$
u = velocity (m/s) = ML^{-1}
d = length (m) = L
F = newtons (kg m /s^2) = MLT^{-2}

For π_1

$$0 = \left(ML^{-3}\right)^{a_1} \left(LT^{-1}\right)^{b_1} (L)^{c_1}\, ML^{-1}T^{-1}$$
$$0 = a_1 + 1$$
$$0 = -3a_1 + b_1 + c_1 - 1$$
$$0 = -b_1 + 1$$
$$a_1 = -1$$
$$b_1 = -1$$
$$c_1 = -1$$
$$\pi_1 = \frac{\mu}{\rho u d}$$

For π_2

$$0 = \left(ML^{-3}\right)^{a_2} \left(LT^{-1}\right)^{b_2} (L)^{c_2}\, MLT^{-2}$$
$$0 = a_2 + 1$$
$$0 = -3a_2 + b_2 + c_2 + 1$$
$$0 = -b_2 - 2$$
$$a_2 = -1$$
$$b_2 = -2$$
$$c_2 = -2$$
$$\pi_2 = \frac{F}{\rho u^2 d^2}$$

$$0 = \phi(\pi_1, \pi_2)$$
$$0 = \phi\left(\frac{u}{\rho u d}, \frac{F}{\rho u^2 d^2}\right)$$

Inverting π_1 gives $\quad \mathrm{Re} = \frac{\rho u d}{\mu}$

$$0 = \phi\left(\mathrm{Re}, \frac{F}{\rho u^2 d^2}\right)$$

Rearranging this gives $F = \rho u^2 d^2 \phi(\text{Re})$

6.c

For dynamic similarity the Reynolds numbers are equal for both the water and air situation

$$\text{Re}_{=} = \text{Re}_{\text{water}}$$

$$\left(\frac{\rho u d}{\mu}\right)_{\text{air}} = \left(\frac{\rho u d}{\mu}\right)_{\text{water}}$$

$$\frac{12.5 \times u_{\text{air}} \times 1.0}{1.7 \times 10^{-5}} = \frac{1000 \times 5.0 \times 0.1}{1.0 \times 10^{-6}}$$

$$u_{\text{air}} = 680 m / s$$