This question paper consists of ? printed pages, each of which is identified by the Code Number CIVE 140001

Graph Paper?

May/June 2004

Examination for the degree of

BEng/ MEng
Civil Engineering

FLUID MECHANICS

Time allowed: 2 hours

Attempt **4** questions

1. (a) A "U"-tube manometer is being used to measure the pressure difference between two points on a horizontal pipe. The fluid in the pipe has a relative density of 0.8 and the manometric fluid has a density 13600 kg/m^3. The two readings on the manometer differ by 0.5m. What is the pressure difference measured by the manometer?

[8 marks]

(b) A storage tank has vertical sides and is filled with water to a depth of 2.0m. If the water is covered with a 1.0m thick layer of oil of relative density 0.8, find the resultant force (per unit width) and its line of action on the wall of the tank.

[9 marks]

(c) Water is being fired at 10 m/s from a hose of 50mm diameter into the atmosphere. The water leaves the hose through a nozzle with a diameter of 30mm at its exit. Find the pressure just upstream of the nozzle and the force on the nozzle.

[8 marks]

1.a

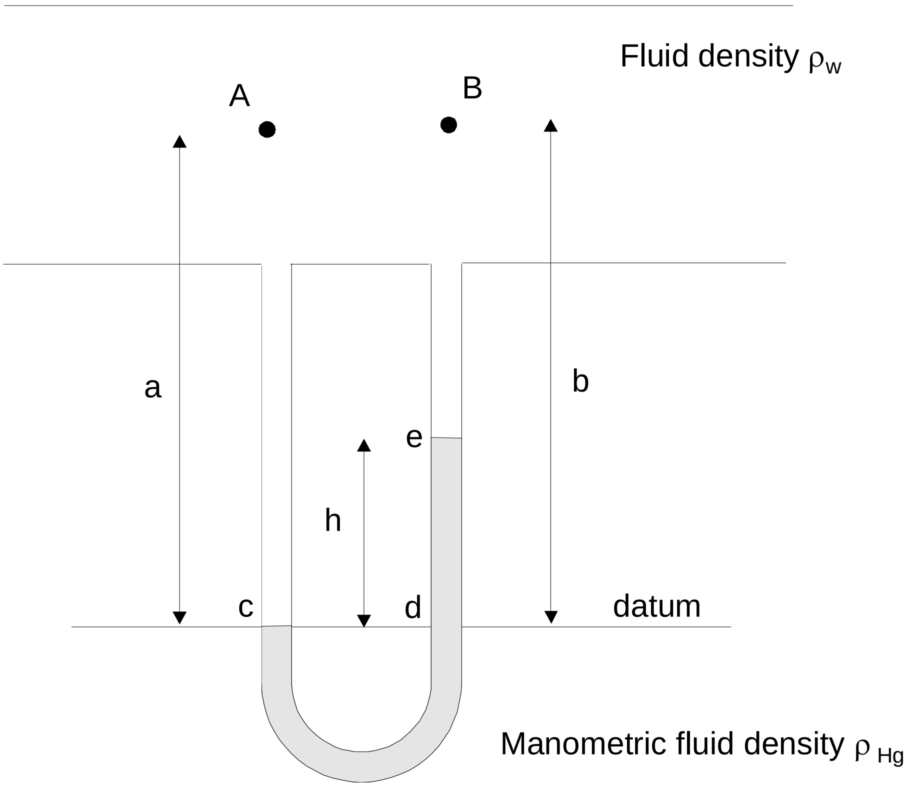


Figure of manometer setup

density of mercury ρ = 13600 kg/m^3

1.a

pressure at C and D is equal:

$p_C = p_D$

$p_A + \rho_w g a = p_B + \rho_w g (b - h) + \rho_{Hg} g h$

$p_A - p_B = \rho_w g b - \rho_w g h - \rho_w g a + \rho_{Hg} g h$

$= \rho_w g (b - a) + hg (\rho_{Hg} - \rho_w)$

As horizontal a = b

$p_A - p_B = hg (\rho_{Hg} - \rho_w)$

$= 0.5 \times 9.81 \times (13600 - 800)$

$= 62\ 784$ N/m^2

$= 62.784$ kN/m^2

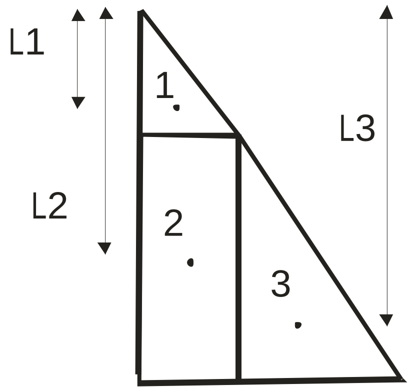
$\rho_o = 800$ kg/m^3

$\rho_w = 1000$ kg/m^3

$h_1 = 1.0$m,

$h_2 = 2.0$m

1.b



Distance of centroids of pressure diagram from surface

$L_1 = 1.0 \times 2/3 = 0.667$ m

$L_2 = 2.0$m

$L_3 = 1.0 + 2 \times 2/3 = 2.333$ m

Areas of pressure diagram

$A_1 = 800 \times 1.0 \times 9.81 \times 1.0 \times 0.5 = 3924$

$A_2 = 800 \times 1.0 \times 9.81 \times 2.0 = 15696$

$A_3 = 1000 \times 2.0 \times 9.81 \times 2.0 \times 0.5 = 19620$

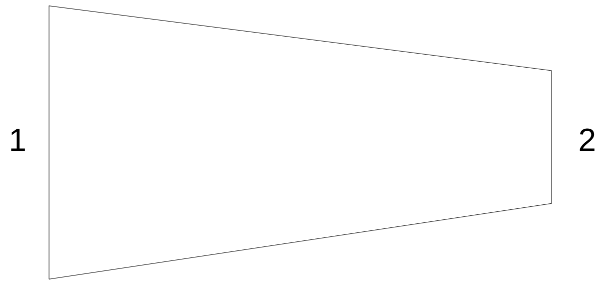
Total area = resultant force per unit width = R = 39240 N

Take moment about surface to find point of action L_R

$R L_R = A_1L_1 + A_2L_2 + A_3L_3 = 79788$ Nm

$L_R = 2.033$ m

1.c



u_1 = unknown
d_1 = 0.05m
u_2 = 10 m/s
d_2 = 0.03m

$A_1 = 0.001963\ m^2$
$A_2 = 0.000707\ m^2$

Use continuity to calculate the unknown velocity

$u_1 = u_2 A_2 / A_1 = 3.6$ m/s

Force on the wall = R = -F = -45.2 N (in the direction of the jet)

Pressure force
Use Bernoulli to calculate the unknown pressure

$$\frac{p_1}{\rho g} + \frac{u_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{u_2^2}{2g} + z_2$$

As horizontal $z_1 = z_2$
p_2 = atmospheric = 0.0

$$p_1 + \rho\frac{u_1^2}{2} = p_2 + \rho\frac{u_2^2}{2}$$

$$\begin{aligned} p_1 &= \frac{\rho}{2}\left(u_2^2 - u_1^2\right) \\ &= \frac{1000}{2}\left(10^2 - 3.6^2\right) \\ &= 43520\, N/m^2 \\ &= 43.5\, kN/m^2 \end{aligned}$$

$$F_P = P_1A_1 - P_2A_2 = 85.45N$$
$$F_T = F_R + F_P + F_B$$
$$F_R = 45.23 - 85.45 = -40.21N$$

2.a A concrete dam has the cross-sectional profile shown in Figure 1. Calculate the magnitude, direction and position of action of the resultant force exerted by the water per unit width of dam?

(15 marks)

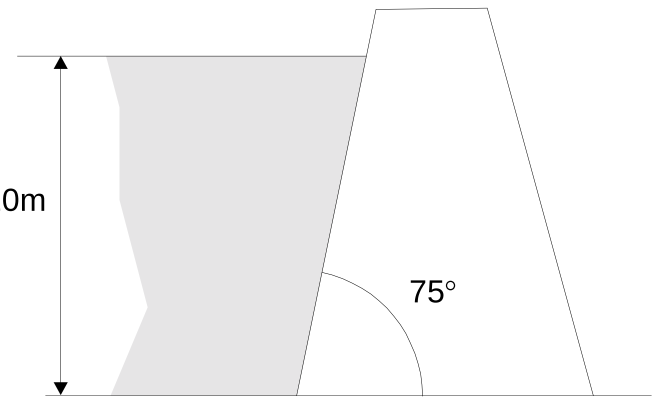


Figure 1

2.b A second design for the same dam has the cross-sectional profile composed of a vertical face with a circular curved section at the base as shown in Figure 2. Calculate the resultant force and its direction of application per unit width of this dam.

(10 marks)

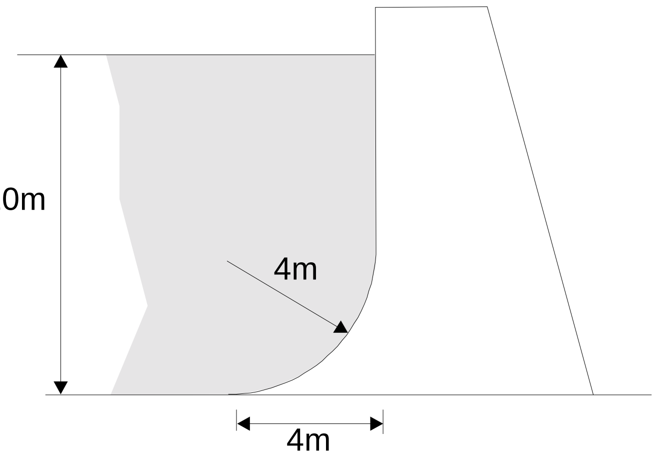


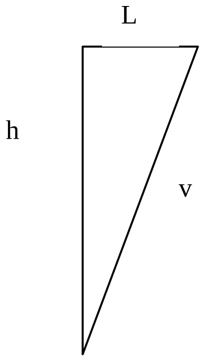
Figure 2

2.a.

<u>Method 1</u>

Vertical force = weight of water = ρ g A b
Horizontal force = force on a projection of the vertical plane

$$= \rho g h \frac{h}{2} b$$



$$L = h \tan\theta = 10 \tan 15 = 2.679m$$

$$A = 0.5hL = 0.5 \times 10 \times 2.679 = 13.397m^2$$

$$Rv = 1000 \times 9.81 \times 13.397 \times 1 = 131425\,N$$

$$Rh = 1000 \times 9.81 \times 10 \times (10/2) \times 1 = 490500\,N$$

$$R = \sqrt{Rv^2 + Rh^2} = 507802\,N$$

Acting at right angle to the wall 15° to the horizontal. Also

$$\tan^{-1}(Rv / Rh) = \phi = 15^\circ$$

Method 2

Force on wall = pressure at centroid × area of wall
= ρg × depth to centroid × area of wall

Sloping wall length, $v = h/\cos 15 = 10.35m$

$$F = 1000 \times 9.81 \times (10.35 \times 1) \times 5$$
$$= 507668\,N$$

Position of this force is through the centre of pressure, Sc.
Using the parallel axis theorem,

$$Sc = \frac{I_{oo}}{A\bar{x}} = \frac{\text{2nd momnt of area}}{\text{1st moment of area}}$$
$$I_{oo} = I_{GG} - A\bar{x}^2$$
$$Sc = \frac{I_{GG}}{A\bar{x}} + \bar{x}$$

$\bar{x}$ is the distance along the face to the centroid = v/2 = 5.175m

$$I_{GG} = \frac{bd^3}{12} = \frac{1 \times 10.35^2}{12} = 92.39$$
$$Sc = \frac{92.39}{10.35 \times 1 \times (10.35 \times 0.5)} + (10.35 \times 0.5)$$
$$= 6.9m$$

This is the distance to the centre of pressure from O.

2.b.

$$b = 1m$$
$$a_1 = 4 \times 6 = 24m^2$$
$$a_2 = \frac{\pi 4^2}{2} = 12.566m^2$$

Vertical force

$$\begin{aligned} Rv &= \text{weight of water} \\ &= \rho g(a_1 + a_2)b \\ &= 1000 \times 9.81 \times (24 + 12.566) \times 1 \\ &= 358712\,N \end{aligned}$$

Horizontal force = force on the projection of vertical plan.
This is the same as in part a of this question.

$$Rh = 490500\,N$$

Resultant force

$$R = \sqrt{Rv^2 + Rh^2} = 607671N$$
$$\tan\phi = \frac{Rv}{Rh}$$
$$\phi = \tan^{-1}\left(\frac{358712}{490500}\right) = 36.178^\circ$$

3a. Derive the following expression that describes flow over a triangular sharp notch weir. Start from the Bernoulli equation and state all assumptions made.

$$Q = C_d \frac{8}{15}\sqrt{2g}\tan\left(\frac{\theta}{2}\right)H^{5/2}$$

3b. A triangular notch weir has a 90° notch and when the head is 8cm the flow is measured at 200 litres/min. What is the coefficient of discharge for this particular notch?

3c. What would be the head over a rectangular notch weir of width 20cm for the same flow rate and coefficient of discharge and the notch in part b?

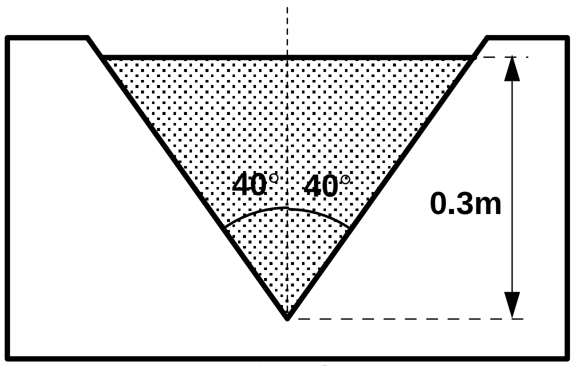


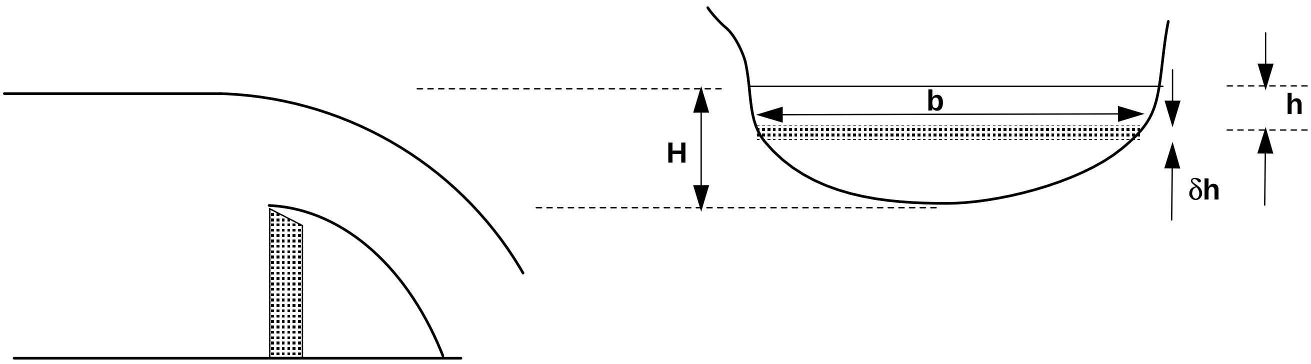
Figure 2

If the level in the tank rises $0.8m$ in 20 seconds, deriving all formulae, determine the coefficient of discharge of the notch.

(20 marks)

A General Weir Equation

Consider a horizontal strip of width *b* and depth *h* below the free surface, as shown in the figure below.



Elemental strip of flow through a notch

Assuming the velocity is only due to the head.

$$\text{velocity through the strip, } \ u = \sqrt{2gh}$$

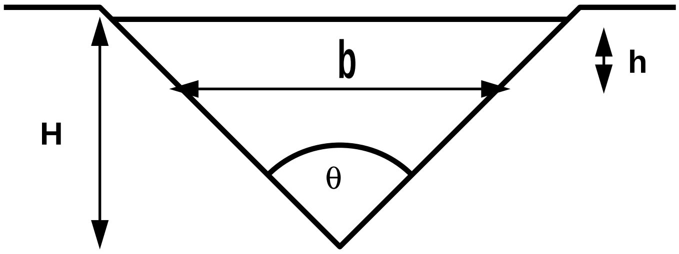
$$\text{discharge through the strip, } \ \delta Q = Au = b\delta h\sqrt{2gh}$$

Integrating from the free surface, $h = 0$, to the weir crest, $h = H$ gives the expression for the total theoretical discharge

$$Q_{\text{theoretical}} = \sqrt{2g}\int_0^H bh^{1/2}\, dh$$

This will be different for every differently shaped weir or notch. To make further use of this equation we need an expression relating the width of flow across the weir to the depth below the free surface.

For the “V” notch weir the relationship between width and depth is dependent on the angle of the “V”.



“V” notch, or triangular, weir geometry.

If the angle of the “V” is θ then the width, *b*, a depth *h* from the free surface is

$$b = 2(H-h)\tan\left(\frac{\theta}{2}\right)$$

So the discharge is

$$\begin{aligned} Q_{\text{theoretical}} &= 2\sqrt{2g}\,\tan\left(\frac{\theta}{2}\right)\int_0^H (H-h)h^{1/2}\,dh \\ &= 2\sqrt{2g}\,\tan\left(\frac{\theta}{2}\right)\left[\frac{2}{5}Hh^{3/2} - \frac{2}{5}h^{5/2}\right]_0^H \\ &= \frac{8}{15}\sqrt{2g}\,\tan\left(\frac{\theta}{2}\right)H^{5/2} \end{aligned}$$

The actual discharge is obtained by introducing a coefficient of discharge

$$Q_{\text{actual}} = C_d\,\frac{8}{15}\sqrt{2g}\,\tan\left(\frac{\theta}{2}\right)H^{5/2}$$

b)

From the question:

Q = 200 litres/min = 3.333 litres/s = 0.00333 m^3/s

θ = 90°

Head = H = 8cm = 0.08m

Re arranging the weir equation, and substituting in these values gives

$$\begin{aligned} C_d &= \frac{Q}{\frac{8}{15}\sqrt{2g}\tan\left(\frac{\theta}{2}\right)H^{5/2}} \\ &= 0.79 \end{aligned}$$

3c

For a rectangular weir of width B the equation is

$$Q_{\text{actual}} = C_d \frac{2}{3} B \sqrt{2g} H^{3/2}$$

From the question

Q = 0.00333 m^3/s

B = 20cm = 0.2m

$$H^{3/2} = \frac{Q}{\frac{2}{3} C_d B \sqrt{2g}}$$

$$= 0.007138$$

$$H = 0.036m$$

$$= 3.6cm$$

END

4 Water flows ate a rate of $0.5 m^3/s$ rising through a 50°, contracting pipe bend. The diameter at the bend entrance is $700\,mm$ and at the exit $500\,mm$ - as shown in Figure 1.

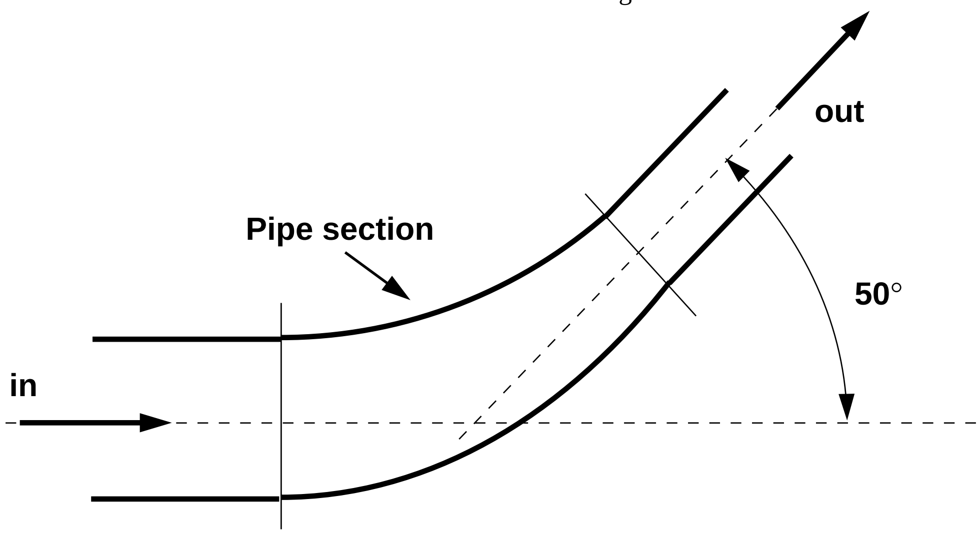


Figure 1.

If the pressure at the entrance to the bend is $200\,kN/m^2$, determine the magnitude and direction of the force exerted by the fluid on the bend. (The exit of the bend is 0.4m higher than the entrance and the bend has a volume of $0.2m^3$.)

(17 marks)

Comment on the reason why frictional losses may be neglected in this analysis.

(3 marks)

$A_1 = \pi d_1 / 4 = 0.3848\ m^2$

$A_2 = \pi d_2 / 4 = 0.1963\ m^2$

$u_1 = Q/A_1 = 0.5/0.3848 = 1.299\ m/s$

$u_2 = Q/A_2 = 0.5/0.1963 = 2.546\ m/s$

$p_1 = 200\ kN/m^2 = 200\ 000\ N/m^2$

<u>Calculate the total force</u>

In the x-direction:

$$
\begin{aligned}
F_{T\,x} &= \rho Q\left(u_{2\,x} - u_{1\,x}\right) \\
u_{1\,x} &= u_1 \\
u_{2\,x} &= u_2 \cos\theta \\
F_{T\,x} &= \rho Q\left(u_2 \cos\theta - u_1\right) \\
&= 1000 \times 0.5(2.546 \cos 50 - 1.299) \\
&= 168.77\ N
\end{aligned}
$$

In the y-direction:

$$
\begin{aligned}
F_{T\,y} &= \rho Q\left(u_{2\,y} - u_{1\,y}\right) \\
u_{1\,y} &= u_1 \sin 0 = 0 \\
u_{2\,y} &= u_2 \sin\theta \\
F_{T\,y} &= \rho Q u_2 \sin\theta \\
&= 1000 \times 0.5 \times 2.546 \sin 50 \\
&= 975.17\ N
\end{aligned}
$$

Calculate the pressure force

Use Bernoulli to calculate force at exit, p_2

$$\frac{p_1}{\rho g} + \frac{u_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{u_2^2}{2g} + z_2 + h_f$$

the friction loss h_f can be ignored, $h_f=0$

As the exit of the pipe is 0.4m higher than the entrance we can say $z_1 = 0.0$, $z_2 = 0.4$

By continuity, $Q= u_1A_1 = u_2A_2$

$$\begin{aligned} p_2 &= p_1 - \frac{\rho}{2}\left(u_2^2 - u_1^2\right) + \left(z_1 - z_2\right) \\ &= 200000 - \frac{1000}{2}\left(2.546^2 - 1.299^2\right) + \left(0 - 0.4\right) \\ &= 193677N \end{aligned}$$

$$F_P = \text{pressure force at 1 - pressure force at 2}$$

$$\begin{aligned} F_{Px} &= p_1A_1\cos 0 - p_2A_2\cos\theta = p_1A_1 - p_2A_2\cos\theta \\ &= 52524N \\ F_{Py} &= p_1A_1\sin 0 - p_2A_2\sin\theta = -p_2A_2\sin\theta \\ &= -29131N \end{aligned}$$

Calculate the body force

$$\begin{aligned} F_{Bx} &= 0 \\ F_{By} &= \rho g \times Volume = 1000 \times 9.81 \times 0.2 = -1962N \end{aligned}$$

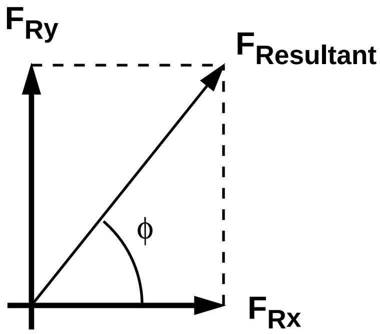
Calculate the resultant force

$$\begin{aligned} F_{Tx} &= F_{Rx} + F_{Px} + F_{Bx} \\ F_{Ty} &= F_{Ry} + F_{Py} + F_{By} \end{aligned}$$

$$\begin{aligned} F_{Rx} &= F_{Tx} - F_{Px} - F_{Bx} \\ &= \rho Q\left(u_2\cos\theta - u_1\right) - p_1A_1 + p_2A_2\cos\theta \\ &= 168.7 - 52524 \\ &= -52356N \end{aligned}$$

$$\begin{aligned} F_{Ry} &= F_{Ty} - F_{Py} - F_{By} \\ &= \rho Q u_2\sin\theta + p_2A_2\sin\theta + \rho g \times Vol \\ &= 975 + 29131 + 1962 \\ &= 32068N \end{aligned}$$

And the resultant force on the fluid is given by



$$F_R = \sqrt{F_{Rx}^2 - F_{Ry}^2} = 61396N$$

And the direction of application is

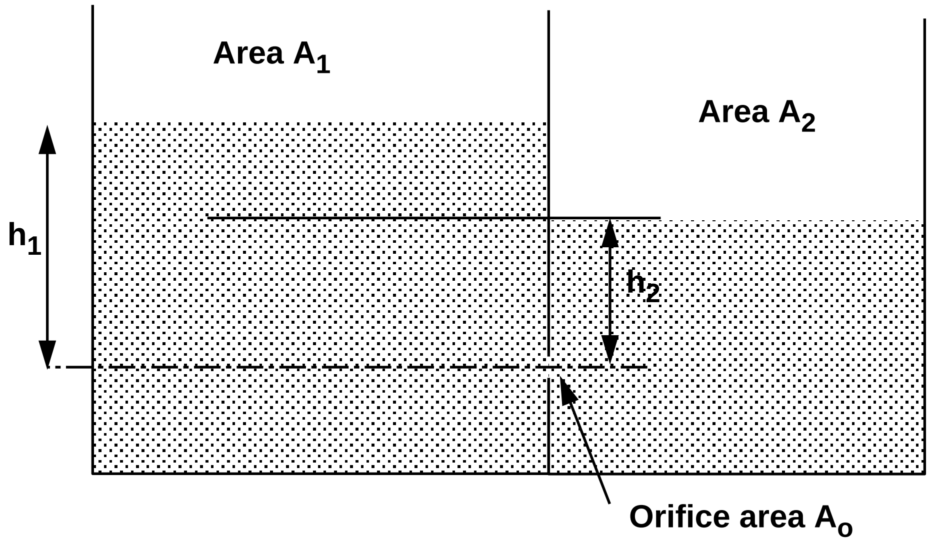
$$\phi = \tan^{-1}\left(\frac{F_{Ry}}{F_{Rx}}\right) = -31.49^\circ$$

the force on the bend is the same magnitude but in the opposite direction

$$R = -F_R$$

5 Two vertical cylindrical tanks of 5m and 3m diameter contain water. They are joined near their bases by a pipe of diameter 5cm which is short enough to be considered an orifice with Cd of 0.6. If the $3m$ diameter tank initially has a level $2m$ higher than the other, calculate how long it will take for the levels to become equal in each tank.

(20 marks)



Two tanks of initially different levels joined by an orifice

Applying the continuity equation

$$Q = -A_1 \frac{\delta h_1}{\delta t} = A_2 \frac{\delta h_2}{\delta t}$$
$$Q\delta t = -A_1 \delta h_1 = A_2 \delta h_2$$

Also we can write $-\delta h_1 + \delta h_2 = \delta h$

So

$$-A_1 \delta h_1 = A_2 \delta h_1 - A_2 \delta h$$
$$\delta h_1 = \frac{A_2 \delta h}{A_1 + A_2}$$

Then we get

$$Q\delta t = -A_1 \delta h_1$$
$$C_d A_o \sqrt{2g(h_1 - h_2)}\ \delta t = \frac{A_1 A_2}{A_1 + A_2} \delta h$$

Re arranging and integrating between the two levels we get

$$\delta t = \frac{A_1 A_2}{(A_1 + A_2) C_d A_o \sqrt{2g}} \frac{\delta h}{\sqrt{h}}$$

$$t = \frac{A_1 A_2}{(A_1 + A_2) C_d A_o \sqrt{2g}} \int_{h_{initial}}^{h_{final}} \frac{\delta h}{\sqrt{h}}$$

$$= \frac{2 A_1 A_2}{(A_1 + A_2) C_d A_o \sqrt{2g}} \left[\sqrt{h}\right]_{h_{initial}}^{h_{final}}$$

$$= \frac{2 A_1 A_2}{(A_1 + A_2) C_d A_o \sqrt{2g}} \left[\sqrt{h_{initial}} - \sqrt{h_{final}}\right]$$

h in this expression is the *difference* in height between the two levels ($h_2 - h_1$).

To get the time for the levels to equal use $h_{initial} = h_1$ and $h_{final} = 0$.

The question says $h_{initial}$ = 2m and we want the time for the tanks to equal so, h_{final} = 0

$A_1 = \pi\, 5^2 /4 = 19.6\ m^2$

$A_2 = \pi\, 3^2 /4 = 7.07\ m^2$

$A_o = \pi\, 0.05^2 /4 = 0.0019634\ m^2$

$$t = \frac{2 A_1 A_2}{(A_1 + A_2) C_d A_o \sqrt{2g}} \left[\sqrt{h_{initial}} - \sqrt{h_{final}}\right]$$

$$= 825.12 \sec$$

$$= 13.752 \min$$

6.a Explain what is meant when we say that equations should be *dimensionally homogenous*.

(4 marks)

6.b Describe the meaning of the terms *dynamic similarity*, *kinematic similarity* and *geometric similarity*.

(9 marks)

6.c Assuming the drag force, F, exerted on a body is a function of the following

fluid density ρ
fluid viscosity μ
diameter d
velocity u

Show that the drag force can be expressed as

$$F = d^2 u^2 \rho \, \phi(\text{Re})$$

where ϕ is some unknown function and Re is the Reynolds number.

(7 marks)

6.a

When giving numerical values to a term in an equation the units in the left side should equal the units on the right side – or more correctly the dimensions should be equal.

Any physical situation can be described by certain familiar properties e.g. length, velocity, area, volume, acceleration etc. These are all known as dimensions.

Dimensions are of no use without a magnitude being attached. We must know more than that something has a length. It must also have a standardised unit - such as a metre, a foot, a yard etc.

Dimensions are properties which can be measured. Units are the standard elements we use to quantify these dimensions.

In dimensional analysis we are only concerned with the nature of the dimension i.e. its quality not its quantity. The following common abbreviation are used:
length = L
mass = M
time = T
force = F
temperature = Θ

We can represent all the physical properties we are interested in with L, T and one of M or F (F can be represented by a combination of LTM).

6.b.

Dimensional analysis is used when constructing physical models of prototype structures. Physical models are used when the fluid flow is particularly complex and difficult to analyse by other means. It enables physical measurements - forces, velocities etc. - taken from the scale models to be converted to the equivalent measurement which would be found on a prototype.

The term similarity relates to physical a scale models.
Geometric similarity - all dimensions are in the in the same ratio.
Dynamic similarity - all velocities are in the same ratio - requires geometric similarity
Kinematic similarity - all forces are in the same ration - requires dynamic similarity.

6.c.

$$F = f(\rho, \mu, d, u)$$
$$0 = \phi(F, \rho, \mu, d, u)$$

Assume the governing variables *ρ, u, d*

According to Buckingham's π theorem there are n-m groups
where

n = number of variables (5) and
m = number of dimensions (i.e. MLT, giving 3)
n-m = 5-3 = 2 groups

$$0 = \phi(\pi_1, \pi_2)$$
$$\pi_1 = \rho^{a_1} u^{b_1} d^{c_1} \mu$$
$$\pi_2 = \rho^{a_2} u^{b_2} d^{c_2} F$$

Dimensions of the variables are:

ρ = density (kg/m³) = ML^{-3}
μ = viscosity (kg/m/s) = $ML^{-1}\,T^{-1}$
u = velocity (m/s) = ML^{-1}
d = length (m) = L
F = newtons (kg m /s²) = MLT^{-2}

For π_1

$$0 = \left(ML^{-3}\right)^{a_1} \left(LT^{-1}\right)^{b_1} \left(L\right)^{c_1} ML^{-1}T^{-1}$$
$$0 = a_1 + 1$$
$$0 = -3a_1 + b_1 + c_1 - 1$$
$$0 = -b_1 + 1$$
$$a_1 = -1$$
$$b_1 = -1$$
$$c_1 = -1$$
$$\pi_1 = \frac{\mu}{\rho u d}$$

For π_2

$$0 = \left(ML^{-3}\right)^{a_2} \left(LT^{-1}\right)^{b_2} \left(L\right)^{c_2} MLT^{-2}$$
$$0 = a_2 + 1$$
$$0 = -3a_2 + b_2 + c_2 + 1$$
$$0 = -b_2 - 2$$
$$a_2 = -1$$
$$b_2 = -2$$
$$c_2 = -2$$
$$\pi_2 = \frac{F}{\rho u^2 d^2}$$

$$0 = \phi(\pi_1, \pi_2)$$
$$0 = \phi\left(\frac{u}{\rho u d}, \frac{F}{\rho u^2 d^2}\right)$$

Inverting π_1 gives $\mathrm{Re} = \dfrac{\rho u d}{\mu}$

$$0 = \phi\left(\mathrm{Re}, \frac{F}{\rho u^2 d^2}\right)$$

Rearranging this gives $F = \rho u^2 d^2 \phi(\mathrm{Re})$

END

END