This question paper consists of 5 printed pages,
(including the formula sheet)
each of which is identified by the Code Number CIVE 140001

Formula Sheet attached

May/June 2006

Examination for the degree of

BEng/ MEng
Civil Engineering

FLUID MECHANICS

Time allowed: 2 hours

Attempt **4** questions

SOLUTIONS

1.

(a) Water is flowing over a sharp-crested rectangular weir of width 35cm into a cylindrical tank of diameter 75cm. In 20 seconds the depth of water in the tank rises 1.2m. Assuming a discharge coefficient of 0.9, determine the height of the water above the weir in mm.

[9 marks]

(b) If the discharge remains the same and the rectangular weir is replaced by a 90^{o} notch weir with a coefficient of discharge of 0.8 and a maximum height of 18cm. Would this notch be adequate to measure this discharge?

[6 marks]

(c) In an experiment a jet of water of diameter 20mm is fired vertically upwards at a sprung target that deflects the water at an angle of 120° to the horizontal in all directions. If a 500g mass placed on the target balances the force of the jet, was is the discharge of the jet in litres/s?

[10 marks]

(a)

Rectangular weir equation

$$Q = C_d \frac{2}{3} B\sqrt{2g} H^{3/2}$$

Calculate the discharge

$$Q = \frac{vol}{time} = \frac{(\pi d^2/4)\times h}{time} = \frac{(\pi 0.75^2/4)\times 1.2}{20} = 0.0265\, m^3/s$$

Substitute in the weir equation

$$0.0265 = 0.9\frac{2}{3}0.35\sqrt{2g}H^{3/2}$$

$$H^{3/2} = 0.0284$$

$$H = 0.093\ m = 93\, mm$$

(b) Triangular notch weir equation

$$Q = C_d \frac{8}{15}\sqrt{2g}\tan\left(\frac{\theta}{2}\right)H^{5/2}$$

Substitute values in this equation to find the head H

$$0.0265 = 0.8\frac{8}{15}\sqrt{2g}\tan\left(\frac{90}{2}\right)H^{5/2}$$

$$H^{5/2} = 0.014$$

$$H = 0.181 m = 181 mm = 18.1 cm$$

This is higher than the 18cm height of the weir, so the weir is NOT adequate to measure this discharge.

(c) Apply force equation

$$F = Q\rho\left(u_{2y} - u_{1y}\right) = Q\rho\left(u_2 \cos\theta - u_1\right)$$

By continuity

$Q_1 = Q_2 = Q$

The total area of flow is constant for an open jet $A_1 = A_2 = A$

So, $u_1 = u_2 = Q/A$

$$F = Q\rho\left(\frac{Q}{A}\cos 120 - \frac{Q}{A}\right) = -\rho\frac{3}{2}\frac{Q^2}{A}$$

This equates to the force of the mass

$$Mg = \rho\frac{3}{2}\frac{Q^2}{A}$$

$$0.5 \times 9.81 = 1000 \times 1.5 \times \frac{Q^2}{\pi 0.02^2 / 4}$$

$$Q = 0.00101 m^3 / s$$

$$Q = 1.01 litres / s$$

2

(a) (i) Explain what is meant by *dimensional homogeneity* and describe how it may be used.

[2 Marks]

(ii) What is a *non-dimensional* or *dimensionless* number? Some common non-dimensional numbers are given names. State three of these.

[5 marks]

(iii) Describe the term *similarity* and what is required to achieve *geometric similarity*, *kinematic similarity* and *dynamic similarity*.

[5 marks]

(b) It can be shown that the resistance to motion, R, of an object moving in a fluid can be given by the following expression:

$$R = \rho u^2 d \phi\left(\frac{\rho u d}{\mu}\right)$$

(i) In an experiment is being designed for a sphere of 1m travelling in air so that it has dynamic similarity with a sphere of diameter 0.01m travelling in water at 2m/s. What speed must the air be travelling?

[5 marks]

(ii) Under these conditions, if the force measured on the sphere in the air is 14200 N, what would be the force on the 0.1m sphere in the water?

[6 marks]

(iii) Comment on the viability of this experiment.

[2 marks]

$\mu_{water} = 1.0 \times 10^{-6}\ kg/ms$ $\mu_{air} = 1.7 \times 10^{-5}\ kg/ms$

$\rho_{water} = 1000\ kg/m^3$ $\rho_{air} = 1.25\ kg/m^3$

a) see notes

b) i)

Dynamic similarity so the Reynolds numbers must be equal in air and water

$$\mathrm{Re}_{air} = \mathrm{Re}_{water}$$

$$\left(\frac{\rho u d}{\mu}\right)_{air} = \left(\frac{\rho u d}{\mu}\right)_{water}$$

$$\left(\frac{1.25 \times u \times 1.0}{1.7 \times 10^{-5}}\right) = \left(\frac{1000 \times 2.0 \times 0.01}{1.0 \times 10^{-6}}\right)$$

$$u = 272\ m/s$$

(ii) Divide the expressions for force for water and air, the function may be cancelled as the Reynolds numbers are equal, giving

$$\frac{R_{water}}{R_{air}} = \frac{\rho_{water} u_{water}^2 d_{water}}{\rho_{air} u_{air}^2 d_{air}}$$

$$R_{water} = R_{air}\frac{\rho_{water} u_{water}^2 d_{water}}{\rho_{air} u_{air}^2 d_{air}} = 14200\frac{1000 \times 2^2 \times 0.01}{1.25 \times 272^2 \times 1.0} = 6.14\ N$$

(iii) This experiment is not valid as the air speed required is probably un attainable.

3. Water flows horizontally along a 220mm pipeline fitted with a 90^o bend that moves the water vertically upwards. The diameter at the outlet of the bend is 150mm and it is 0.5m above the centreline of the inlet. If the flow through the bend is 150 litres/s and assuming no losses due to friction, calculate the magnitude and direction of the resultant force the bend support must withstand.
The volume of the bend is 0.02m^3 and the pressure at the inlet is 100 kN/m^2

[25 marks]

Solution

$A_1 = \pi d_1 / 4 = 0.0380$ m^2 $A_2 = \pi d_2 / 4 = 0.01767$ m^2 $Q = 150 / 1000 = 0.15$ m^3/s

$u_1 = Q/A_1 = 3.946$ m/s $u_2 = Q/A_2 = 8.488$ m/s $p_1 = 100$ kN/m^2 = 100 000 N/m^2

Calculate the total force

In the x-direction:

$$F_{T\,x} = \rho Q\left(u_{2x} - u_{1x}\right) = \rho Q\left(u_2 \cos\theta - u_1\right)$$
$$= 1000 \times 0.15\left(8.488 \cos 90 - 3.946\right)$$
$$= -591.9 N$$

In the y-direction:

$$F_{T\,y} = \rho Q\left(u_{2y} - u_{1y}\right) = \rho Q u_2 \sin\theta$$
$$= 1000 \times 0.15 \times 8.488 \sin 90$$
$$= 1273.2 N$$

Calculate the pressure force

Use Bernoulli to calculate force at exit, p_2

$$\frac{p_1}{\rho g} + \frac{u_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{u_2^2}{2g} + z_2 + h_f$$

the friction loss h_f can be ignored, $h_f = 0$

As the exit of the pipe is 0.5m higher than the entrance we can say $z_1 = 0.0$, $z_2 = 0.5$

By continuity, $Q = u_1 A_1 = u_2 A_2$

$$p_2 = p_1 - \frac{\rho}{2}\left(u_2^2 - u_1^2\right) + \left(z_1 - z_2\right)\rho g$$
$$= 100000 - \frac{1000}{2}\left(8.488^2 - 3.946^2\right) + 0$$
$$= 66855\ N$$

$$F_P = \text{pressure force at 1 - pressure force at 2}$$

$$F_{P\,x} = p_1 A_1 \cos 0 - p_2 A_2 \cos\theta = p_1 A_1 - p_2 A_2 \cos\theta$$
$$= 3801 N$$
$$F_{P\,y} = p_1 A_1 \sin 0 - p_2 A_2 \sin\theta = -p_2 A_2 \sin\theta$$
$$= -1181 N$$

Calculate the body force

$$F_{Bx} = 0 \qquad F_{By} = -volume \times \rho \times g = -0.02 \times 1000 \times 9.81 = -196.2\ N$$

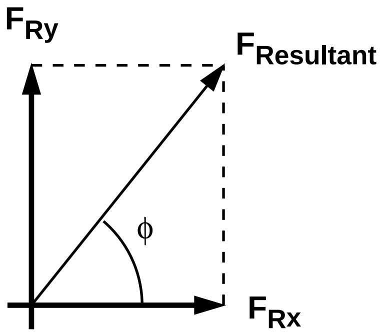
Calculate the resultant force

$$F_{T\,x} = F_{R\,x} + F_{P\,x} + F_{B\,x}$$
$$F_{T\,y} = F_{R\,y} + F_{P\,y} + F_{B\,y}$$

$$F_{Rx} = F_{Tx} - F_{Px} - F_{Bx} = -4393N$$

$$F_{Ry} = F_{Ty} - F_{Py} - F_{By} = 2651N$$

And the resultant force on the fluid is given by



$$F_R = \sqrt{F_{Rx}^2 - F_{Ry}^2} = 5131N$$

And the direction of application is

$$\phi = \tan^{-1}\left(\frac{F_{Ry}}{F_{Rx}}\right) = -31.1^\circ$$

the force on the bend is the same magnitude but in the opposite direction

$$R = -F_R$$

b) The frictional force would be taken into account with a head loss term of the form $h_f = k\frac{u^2}{2g}$ in the Bernoulli equation. i.e.

$$\frac{p_1}{\rho g} + \frac{u_1}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{u_2}{2g} + z_2 + h_f$$

4. (a)

A concrete dam of width 15m has the cross-sectional profile shown in Figure 1. Calculate the magnitude, direction and position of action of the resultant force exerted by the water on the dam.

(15 marks)

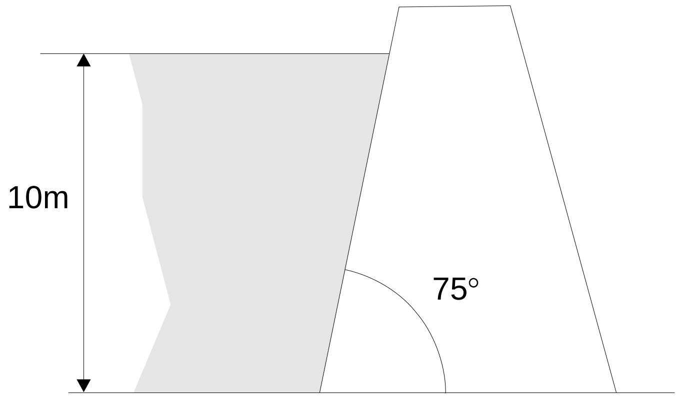


Figure 1

(b) A second design for the same dam has the cross-sectional profile composed of a vertical face with a circular curved section at the base as shown in Figure 2. Calculate the resultant force and its direction of application on this dam design.

(10 marks)

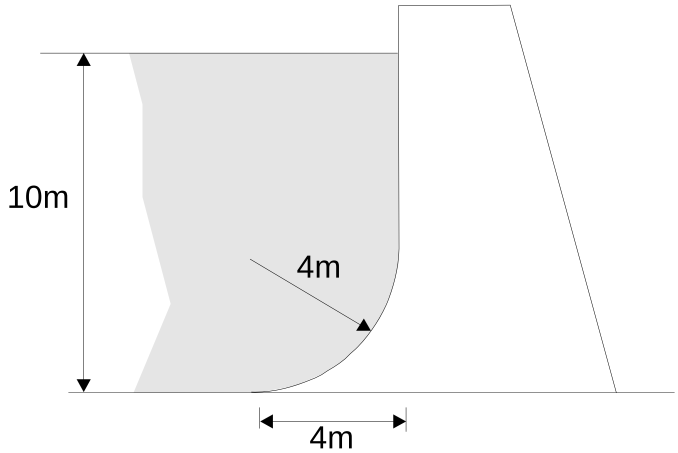


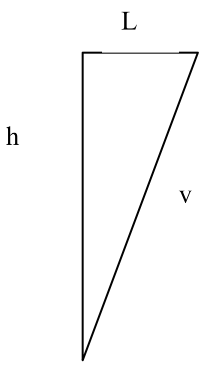
Figure 2

[6 marks]

<u>Method 1</u>

Vertical force = weight of water = ρ g A b
Horizontal force = force on a projection of the vertical plane

$$= \frac{\rho g h^2}{2} b$$



$$L = h \tan\theta = 10 \tan 15 = 2.679 m$$

$$A = 0.5hL = 0.5 \times 10 \times 2.679 = 13.397 m^2$$

$$Rv = 1000 \times 9.81 \times 13.397 \times 15 = 1971368\ N$$

$$Rh = 1000 \times 9.81 \times \frac{10}{2} \times (10 \times 15) = 7357500\ N$$

$$R = \sqrt{Rv^2 + Rh^2} = 7617026\ N$$

Acting at right angle to the wall 15° to the horizontal. Also $\tan^{-1}(Rv/Rh) = \phi = 15^\circ$

Method 2

Force on wall = pressure at centroid × area of wall
= ρg × depth to centroid × area of wall

Sloping wall length, $v = h/\cos 15 = 10.35m$

$$F = 1000 \times 9.81 \times (10.35 \times 20) \times 5$$
$$= 7615012\,N$$

Position of this force is through the centre of pressure, Sc.
Using the parallel axis theorem,

$$Sc = \frac{I_{oo}}{A\bar{x}} = \frac{\text{2nd momnt of area}}{\text{1st moment of area}}$$
$$I_{oo} = I_{GG} - A\bar{x}^2$$
$$Sc = \frac{I_{GG}}{A\bar{x}} + \bar{x}$$

$\bar{x}$ is the distance along the face to the centroid = v/2 = 5.175m

$$I_{GG} = \frac{bd^3}{12} = \frac{15 \times 10.35^3}{12} = 1386$$
$$Sc = \frac{1386}{10.35 \times 15 \times (10.35 \times 0.5)} + (10.35 \times 0.5)$$
$$= 6.9m$$

This is the distance to the centre of pressure from O.

2.b.

$$b = 15m$$
$$a_1 = 4 \times 6 = 24m^2$$
$$a_2 = \frac{\pi 4^2}{2} = 12.566m^2$$

Vertical force

$$Rv = \text{weight of water}$$
$$= \rho g(a_1 + a_2)b$$
$$= 1000 \times 9.81 \times (24 + 12.566) \times 15$$
$$= 5380869\,N$$

Horizontal force = force on the projection of vertical plan.
This is the same as in part a of this question.

$$Rh = 7357500\,N$$

Resultant force

$$R = \sqrt{Rv^2 + Rh^2} = 9115075\,N$$
$$\tan\phi = \frac{Rv}{Rh}$$
$$\phi = \tan^{-1}\left(\frac{5380869}{7357500}\right) = 36.18^\circ$$

5. Describe with the aid of diagrams the following:

   (i) Newton's Law of Viscosity

   [5 marks]

   (ii) The laminar boundary layer

   [5 marks]

   (iii) The turbulent boundary layer

   [5 marks]

   (iv) Boundary layer separation

   [5 marks]

   (v) Methods to prevent boundary layer separation

   [5 marks]

As notes.

6. **(a)** Starting with the Bernoulli and Continuity equations, show that the following expression gives the discharge measured by a venturimeter.

$$Q = C_d A_1 A_2 \sqrt{\frac{2g\left(\frac{p_1 - p_2}{\rho g} + z_1 - z_2\right)}{A_1^2 - A_2^2}}$$

and that when a manometer is attached the discharge may be given by

$$Q_{actual} = C_d A_1 A_2 \sqrt{\frac{2gh\left(\frac{\rho_{man}}{\rho} - 1\right)}{A_1^2 - A_2^2}}$$

[7 marks]

(b) A vertical venturimeter is being used to measure the flow of oil of relative density 0.88 in a 200mm diameter pipe. The throat diameter of the venturimeter is 100mm and the discharge coefficient is 0.96. Two tapping point at the throat and entrance are 320mm apart and the pressure difference between these is measured at 28 kN/m^2. What is the discharge and the velocity in the pipe?

If a mercury manometer were attached at the tapping points what would be the difference in levels of the two arms of the manometer?
[Assume the relative density of mercury is 13.6.]

[8 marks]

(c) The velocity of the water flowing in the same pipe is also measured using a pitot-static tube located centrally in the flow. If the height measured on the attached mercury manometer is 15mm, determine the velocity of the oil.

[8 marks]

(d) Explain why the velocity measured by the pitot-static tube is higher than that measured by the venturimeter.

[2 marks]

a) as notes
b) + c)
To calculate the discharge us this equation

$$Q_{actual} = C_d A_1 A_2 \sqrt{\frac{2g\left[\frac{p_1 - p_2}{\rho g} + z_1 - z_2\right]}{A_1^2 - A_2^2}}$$

d_1 = 0.2m, A_1 = 0.0314 m^2
d_2 = 0.1m, A_2 = 0.00785 m^2
Cd = 0.96
$p_1 - p_2$ = 28000 N/m^2
$z_1 - z_2$ = -0.32 m
ρ = 880 kg/m^3

Substitue in the above equation to give
Q = 0.059 m^3/s
Velocity = Q/A_1 = 1.88 m/s

Equating the expressions given in the question for Q gives

$$C_d A_1 A_2 \sqrt{\frac{2g\left(\frac{p_1 - p_2}{\rho g} + z_1 - z_2\right)}{A_1^2 - A_2^2}} = C_d A_1 A_2 \sqrt{\frac{2gh\left(\frac{\rho_{man}}{\rho} - 1\right)}{A_1^2 - A_2^2}}$$

This simplifies to

$$\left(\frac{p_1 - p_2}{\rho g} + z_1 - z_2\right) = h\left(\frac{\rho_{man}}{\rho} - 1\right)$$

Substitute in appropriate values to get h

$h = 0.202$ m = 202mm

c) Equation of Pitot tube

$$u_1 = \sqrt{\frac{2gh(\rho_m - \rho)}{\rho}}$$

$$u_1 = \sqrt{\frac{2g0.015(13600 - 880)}{880}} = 2.06\, m/s$$

d) The reason for the difference between the velocity value calculated by the Pitot tube and the Venturimeter is tahteth Pitot tube measures a *point* velocity and the venture a discharge which is converted to a *mean* velocity. The Pitot tune was in the middle of the pipe, where you would expect the largest velocity, hence the measured value is slightly greater than that from the venturimetre.

FORMULA SHEET

$$\tau = \mu \frac{du}{dy} \qquad \nu = \frac{\mu}{\rho} \qquad p = \rho g h$$

$$R = \rho g \bar{z} A$$

R = pressure at centroid × area

$$S_c = \frac{I_{oo}}{A\bar{x}}$$

$$I_{oo} = I_{GG} + A\bar{x}^2 \qquad Q = Au = A_1 u_1 = A_2 u_2$$

$$\frac{p_1}{\rho g} + \frac{u_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{u_2^2}{2g} + z_2 + h_f$$

$$u = \sqrt{2g(h_2 - h_1)} \qquad u_1 = \sqrt{\frac{2gh(\rho_m - \rho)}{\rho}} \qquad h_f = \frac{32 \mu L u}{\rho g d^2}$$

$$Q_{actual} = C_d A_1 A_2 \sqrt{\frac{2gh\left(\frac{\rho_{man}}{\rho} - 1\right)}{A_1^2 - A_2^2}} \qquad Q = C_d A_o \sqrt{2gh} \qquad Q_{\text{theoretical}} = \sqrt{2g} \int_0^H b h^{1/2}\, dh$$

$$Q_{actual} = C_d A_1 A_2 \sqrt{\frac{2g\left[\frac{p_1 - p_2}{\rho g} + z_1 - z_2\right]}{A_1^2 - A_2^2}} \qquad Q = C_d \frac{2}{3} B \sqrt{2g} H^{3/2} \qquad Q = C_d \frac{8}{15} \sqrt{2g} \tan\left(\frac{\theta}{2}\right) H^{5/2}$$

$$F_T = F_R + F_B + F_P \qquad \mathrm{Re} = \frac{\rho u d}{\mu} \qquad Q = \frac{\Delta p}{L} \frac{\pi d^4}{128 \mu}$$

$$F = Q\rho(u_2 - u_1)$$

End of Paper