

CIVE1400, Fluid Mechanics

Marked Problem Sheet

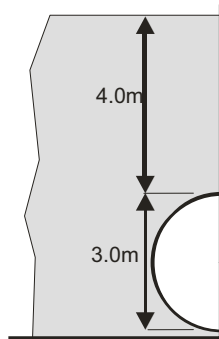
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This work must be submitted via the print room by 4.00pm
Tuesday 24th April 2007
No marks will be given for work submitted after this date.

You will receive the marked work complete with solutions by May 8th 2007.

Question 1.

- (a) For the circular gate shown in figure 1, submerged in water, what is the minimum weight (per unit length) of the gate so that it does not float.



- (b) The face of a dam is vertical to a depth of 10m below the water surface then slopes at 35° to the vertical. If the depth of water is 20m what is the resultant force per metre acting on the whole face?
- (c) Water flows in a circular pipe which reduces in diameter from 500mm at point A to 300mm at point B. Then pipe then splits into two branches of diameters 0.25m and 0.2m discharging at C and D respectively. If the velocity at A and D is 1.0m/s, what is the discharge at C and D and the velocity at B and C?
- (d) If point A is 10m higher than point B and the pressure at A is 10kN/m², what is the pressure at point B?

[12 Marks]

Question 2.

The maximum depth of water in a reservoir is maintained by a V-notch weir, as shown in figure 1. When the water exceeds its optimum depth, the excess discharges via the weir into a spillway.

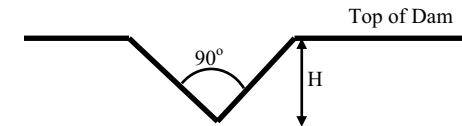


Figure 1. Notch weir geometry.

Experience has shown that under storm conditions this weir is not adequate to cope with the excess flow and needs to be modified.

- (a) If the maximum discharge, above which water will start to flow over the dam, is 25000 m³/hr, determine the height of the weir if its discharge coefficient is 0.9.
- (b) The first proposed new design is to increase the angle of the notch to 120°. Determine the maximum hourly discharge over this weir if the height of the weir remains the same and the discharge coefficient is 0.85.
- (c) A second proposed design is to replace the notch weir with a rectangular weir of width 200cm and the same height as the original weir. Determine the maximum discharge over this weir if $C_d = 0.85$.
- (d) The new weir design must allow a peak flow at least 40% greater than the original weir design. Using this criterion, which of the two weirs would you choose?

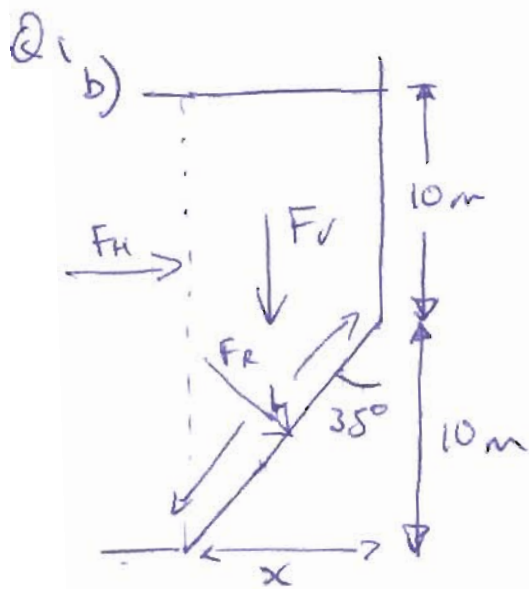
[8 Marks]

Question 3.

A pipe which is carrying water turns through 180° in the horizontal plane, at the same time it reduces diameter from 0.5m at the entrance to the bend to 0.3m at the exit. The water is measured as flowing at the rate of 500 litres/s with a pressure at the entrance of 1.45 bar.

Neglecting any loss in head for friction, calculate the force exerted by the water on the bend, and its direction of application.

[10 Marks]



There are various ways to tackle this. One is to treat the problem in a similar way to a curved surface

Horizontal force $F_H = \text{pressure at centroid} \times \text{Area}$

$$= \rho g \frac{H}{2} \times H \times 1$$

$$H = 20 \text{ m}$$

$$= 1,962,000 \text{ N/m}$$

Vertical force $F_v = \text{weight of water above surface}$

$$= \left(10 \times x + \frac{10 \times x}{2} \right) \rho g$$

$$\tan 35 = \frac{x}{10} \quad x = 7.0 \text{ m}$$

$$F_v = 1,030,050 \text{ N/m}$$

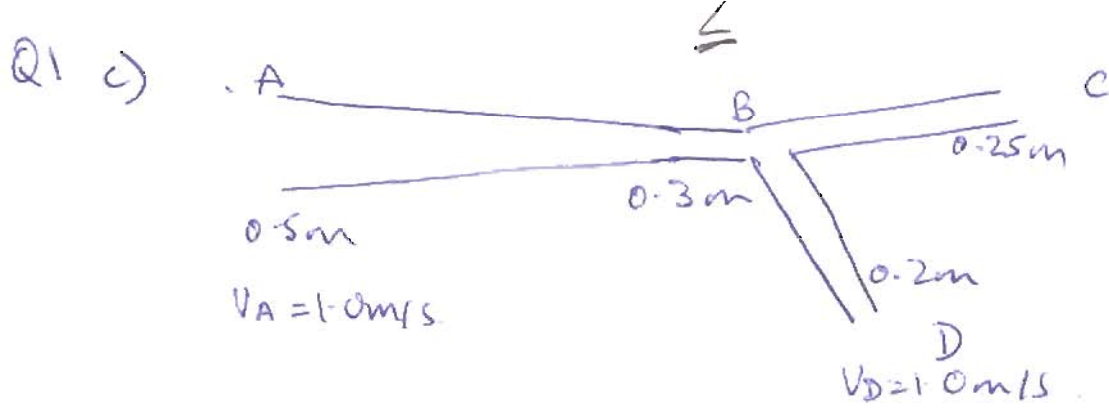
$$\begin{aligned} \text{Resultant} &= \sqrt{F_v^2 + F_H^2} \\ &= 2,215,953 \text{ N/m} \end{aligned}$$

The force on sloping face = $(10 + 5) \rho g \times L \times 1$.

$$L = 12.2 \text{ m}$$

$$F_R = 1,795,230 \text{ N/m}$$

The question did not ask for this calculation.



$$A_A = 0.196 \text{ m}^2$$

$$A_B = 0.0709 \text{ m}^2$$

$$A_C = 0.049 \text{ m}^2$$

$$A_D = 0.031 \text{ m}^2$$

$$Q_B = Q_A = A_A V_A = 0.196 \text{ m}^3/\text{s}$$

$$Q_D = A_D V_D = 0.031 \text{ m}^3/\text{s}$$

$$Q_C = Q_A - Q_D = 0.165 \text{ m}^3/\text{s}$$

$$V_B = \frac{Q_B}{A_B} = 2.76 \text{ m/s}$$

$$V_C = \frac{Q_C}{A_C} = 3.36 \text{ m/s}$$

d) Bernoulli

$$\frac{P}{\rho g} + \frac{V_A^2}{2g} + z_A = \frac{P_B}{\rho g} + \frac{V_B^2}{2g} + z_B$$

$$z_A - z_B = 10 \text{ m}$$

$$\frac{10000}{\rho g} + \frac{1}{2g} + 10 - \frac{2.76^2}{2g} = \frac{P_B}{\rho g}$$

$$10.682 = P_B / \rho g$$

$$P_B = 104791 \text{ N/m}^2$$

Q2

$$V_{notch} \cdot Q = C_d \frac{8}{15} \sqrt{2g} \tan\left(\frac{\theta}{2}\right) H^{5/2}$$

$$\text{rectangular } Q = C_d \frac{2}{3} B \sqrt{2g} H^{3/2}$$

a) $Q_{max} = 25000 \text{ m}^3/\text{hr}$

$$Q_{max} = \frac{25000}{60 \times 60} = 6.944 \text{ m}^3/\text{s}$$

$$C_d = 0.9$$

$$\theta = 90^\circ$$

using Vnotch equation

$$H^{5/2} = 3.266$$

$$H = 1.61 \text{ m}$$

b) $\theta = 120^\circ$ $H = 1.61 \text{ m}$

$$C_d = 0.85$$

$$Q = 11.36 \text{ m}^3/\text{s}$$

$$Q = 40896 \text{ m}^3/\text{hr}$$

c) $B = 2.0 \text{ m}$

$$C_d = 0.85$$

$$H = 1.61 \text{ m}$$

rectangular

$$Q = 10.21 \text{ m}^3/\text{s}$$

$$Q = 36765 \text{ m}^3/\text{hr}$$

d) design flow = $6.944 \times 1.4 = 9.72 \text{ m}^3/\text{s}$

Either design will work.

The rectangular is probably a little better as it is closer and not "over designed".

$$Q3 \quad Q = 500 \text{ L/s} = 0.5 \text{ m}^3/\text{s}$$

$$A_1 = 0.196 \text{ m}^2$$

$$u_1 = Q/A_1 = 2.546 \text{ m/s}$$

$$A_2 = 0.071 \text{ m}^2$$

$$u_2 = Q/A_2 = 7.074 \text{ m/s}$$

$$\theta = 180^\circ$$

$$P_1 = 1.45 \text{ bar} = 1.45 \times 10^5 \text{ N} = 145000 \text{ N}$$

Total Force

$$F_{\text{Total}} = \rho Q (u_2 \cos 180 - u_1) = -4810 \text{ N}$$

$$F_{\text{Total } y} = \rho Q (u_2 \sin 180 - u_1 \sin 0) = 0$$

Pressure Force

from Bernoulli $\frac{P_1}{\rho g} + \frac{u_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{u_2^2}{2g} + Z_2 + h_f$

$$h_f = 0$$

$$Z_1 = Z_2 \text{ (horizontal)}$$

$$P_2 = P_1 - \rho (u_2^2 - u_1^2) = 123224 \text{ N/m}^2$$

$$F_{\text{Px}} = P_1 A_1 - P_2 A_2 \cos 180 = 37181 \text{ N}$$

$$F_{\text{Py}} = -P_2 A_2 \sin 180 = 0$$

Body force

$$F_{\text{Bx}} = F_{\text{By}} = 0 \text{ as horizontal}$$

Resultant Force

$$F_{\text{Tx}} = F_{\text{Rx}} + F_{\text{Px}} + F_{\text{Bx}} \quad F_{\text{Tx}} = -41991 \text{ N}$$

$$F_{\text{Ty}} = 0$$

$$\text{Resultant} = -41991 \text{ N}$$

$$\text{Angle} = -180^\circ$$